IPO Pricing Strategies with Deadweight and Search Costs

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Abstract
The model in this paper provides a complementary explanation to the well-known IPO pricing puzzle. The model allows the investor to make a decision on whether and when information should be gathered, and allows a purchase decision based on the information. With this investor’s decision-making process in mind, firms price their IPOs to maximize their payoffs by trying to avoid an IPO failure and by assessing the investor’s possible post-search outcomes. While the model provides implications to the general IPO puzzle, the results seem particularly relevant for explaining the pricing of REIT IPOs, MLP IPOs, and mutual fund IPOs. The model may also help explain why IPO underpricing levels change over time and suggests that underpricing levels might vary across industries.

One of the most puzzling and much studied empirical phenomena in the finance literature is probably the disparity between the offering prices of initial public offerings (IPOs) and their market clearance prices on the first day of trading. It is estimated that investors in the IPO market earn an average initial day return of 17.3% from the 7,597 IPOs issued during the 1975–2005 period.1 This positive and significant initial day return is consistent over sub-periods but seems to be higher for more recent IPOs. This pricing puzzle is probably difficult to explain because the process involves several different types of agents with conflicting interests and different information. In addition, complicated regulatory constraints might also have made this problem more time- and location-specific.

While many theorists and empiricists have attempted to address this issue from various facets, the most developed area so far is that linked to stories based on asymmetric information between issuers and underwriters; between issuers and investors; between underwriters and investors; and among underwriters, uninformed investors, and informed investors. The main explanations using the asymmetric information story include: (1) keeping uninformed investors in the market, (2) encouraging information production and revelation, and (3) signaling a firm’s quality.2 Other fruitful avenues that help explain the IPO puzzle include legal liability, aftermarket trading activities (including price stabilization and flipping), investor sentiment, and investor behavior.3 However, Ritter and Welch
Chan, Wang, and Yang (2002) seem to convincingly conclude that asymmetric information cannot be the primary driver of the many IPO phenomena documented in the literature. Recently, more emphasis seems to be placed on the darker side of the issues that are related to agency problems between underwriters and issuers, allocation strategies used by underwriters, and the possibility that IPO investors are deceived by analysts’ aggressive growth forecasts.4

It might be fair to say that while most of the theories developed so far have received some empirical support (at varied levels), none of them seem to be able to fully justify the pricing phenomenon. Furthermore, some of the empirical evidence on IPOs cannot be adequately explained by any of the existing theories. For example, none of the existing theories can fully explain why Real Estate Investment Trust (REIT) IPOs were overpriced during the 1971–1988 period (and slightly underpriced in the 1990–2000 period) and why Master Limited Partnership (MLP) and mutual fund IPOs are not underpriced.5 In addition, the large magnitude of the underpricing of high technology stock IPOs seems difficult to justify using rational explanations.

Among all the agents (owners of the firm, executives of the firm, underwriters, informed investors, and uninformed investors) involved in the IPO market, the least examined agent is probably the issuer (owner of the firm). Baron (1982) first argues that underpricing could be due to the information asymmetry between the underwriter and the firm. The empirical evidence on this issue is mixed. Muscarella and Vetsuypens (1989) demonstrate that underwriters underprice their own IPOs as much as other IPOs in the market, while Ljungqvist and Wilhelm (2003) report a negative relationship between the initial day return and the investment bank’s equity holding. Allen and Faulhaber (1989), Grinblatt and Hwang (1989), Welch (1989), Jegadeesh, Weinstein, and Welch (1993), and Welch (1996) establish a convincing literature suggesting that issuers are willing to burn money in the IPO market in order to signal their quality. More recent research on issuer’s behavior is done by Loughran and Ritter (2002). The authors predict that issuers will care less about the offering price of an IPO (and hence not negotiate aggressively with their underwriters) if they expect their stocks to perform well in the aftermarket.

In this paper, we examine the IPO pricing strategies of issuing firms to shed light on the underpricing issue. The intuition of our model is simple and is similar to that of a traveler driving along a country road in the evening looking for a hotel to stay for the night. The traveler knows there are two hotels available. One hotel is about five miles away on the left side of the road while the other is about five miles away on the right side of the road. The traveler has general knowledge about the quality and the price range of the hotels from reading a travel guide, but does not know which hotel provides a better deal. To make a decision, the traveler randomly selects a hotel and invests time (and driving effort) to inspect the hotel. After inspecting the first hotel, the traveler will judge if its price is reasonable. She will stay at the hotel if she deems that its price conforms to her expectation. However, if she judges the first hotel’s price to be too high given its quality, she
will have an incentive to drive to the second hotel to check it out. If she decides to drive away, she will have to invest more time (and driving effort). Apparently, there is an incentive for her not to drive away given this additional cost. If, after inspecting the second hotel, she still judges its price to be too high given its quality, she will have to incur further costs to find other alternatives. Thus, the trade-off for the traveler is between incurring more search costs and taking the current offer.

Both hotel owners are aware of the traveler’s decision-making process but do not know which hotel the traveler will visit first. The owner of the hotel first visited by the traveler knows that, if the traveler leaves and then finds the price of the second hotel to be more attractive, he will lose the traveler. But, if the traveler also finds the price offered by the second hotel owner to be too high, the individual will either return to the first hotel or seek other alternatives. Both owners, realizing the possibility that the room will not be sold if they lose the traveler, have an incentive to lower the room rate to a level that they believe the traveler will accept.

Given this process, the pricing decision of each hotel owner involves three parameters. The first is the cost for the traveler to search and inspect another alternative (to drive away). The higher the traveler’s search cost, the lower the owner’s incentive to reduce rates. The second is the owner’s loss of revenue (deadweight costs) if the traveler leaves. Of course, the higher the deadweight loss, the higher the owner’s incentive to lower rates. The last parameter is the range of room rates from which a traveler might draw. Depending on the traveler’s background and prior experience, the room rate that is deemed reasonable may vary among travelers. If the hotel owner believes that the rate the traveler finds acceptable comes from a wide range, then the owner has an incentive to set a high rate. This is the case because, holding everything else constant, the probability for the traveler to leave when offered a high rate is lower when the range is wider. Given this, an owner of a resort hotel may have less incentive to lower the rates (since the rates deemed reasonable by travelers could vary significantly) when compared to an owner of a standard hotel (for which travelers may have a better consensus of opinion on an acceptable rate).

The IPO issuing firms may follow a similar logic in their pricing decisions. Our model includes two firms issuing IPOs simultaneously and one representative investor. The two firms will announce their IPO prices and the investor will do the research. After obtaining information on the first issuing firm that the investor randomly picks, the investor will decide whether to purchase its stock or to research another firm. If the investor decides to do the latter, additional costs will be incurred. If, after the second search the investor still thinks that the prices of both offering firms are too high, other investment opportunities will be investigated. For the issuing firms, we assume there is a deadweight cost (in the form of reputation loss and issuance cost) if the issuance fails. On the other hand, the investor may have an incentive to purchase the stock of the first firm as additional searches will involve additional costs. Given this arrangement, firms have an incentive to price high if the investor’s search cost is high, to price low
if the cost of a failed issuance is high, and to price high if the chance for an investor to come up with a high value is high. Therefore, depending on the parameter values, we could observe an overpricing of IPOs, a slight underpricing of IPOs, or a significant underpricing of IPOs.

One of the unique features of our model is that we do not need to assume one agent has superior information over others. We allow the agents to act based on the information they gather (some with a search cost), even if that information may differ among the agents. This information structure differs from most (if not all) information structures that have been used to explain IPO pricing (or allocation) strategies. With this information structure, we are able to provide a complementary explanation to the other existing theoretical frameworks. While we believe that our model predictions have general implications to the observed IPO pricing puzzle, the results are particularly helpful in explaining why REIT IPOs can be overpriced, and why MLP IPOs and mutual fund IPOs are not under- or over-priced. Furthermore, our model may help offer a partial explanation for the significant underpricing of IPOs during hot markets.

This paper proceeds as follows. Section two presents the basic model. In that section, we only model the interaction between issuers (the firms) and the investor. The underwriter does not play a role. To some extent, we are modeling a best-efforts contract. In section three, we expand the model by bringing the underwriter into the picture and discussing a firm’s pricing strategy under a firm commitment contract. The section also discusses the factors affecting the optimal contract choice. Section four contains our conclusions.

The Basic Model

We assume there is a representative investor, symbolized as $I$, who is looking for an investment opportunity in the IPO market. Meanwhile, there are two firms, $F_i$ and $F_j$. Each of these two firms offer one share of IPO stock for sale, and their intrinsic values are $V_i$ and $V_j$, respectively. We assume that the firms are symmetric in their pricing decisions and do not play games with each other. In this regard, if the firms have identical intrinsic values, their pricing decisions will be the same. We also assume that each firm can estimate the intrinsic values of both its own stock and its rival’s stock, $V_i$ and $V_j$. The investor, on the other hand, is unable to know with certainty the intrinsic values that the firms assign to their stocks. However, there is public information on these firms prior to the investor’s search that the stock value of firm $F_x(x = i, j)$ follows a uniform distribution, or, $V_{bx} \sim U[\bar{V}_x - \xi, \bar{V}_x + \xi]$, with a mean $\bar{V}_x$ and a variance $\frac{1}{3}\xi^2$. We assume that the information on $\bar{V}_x$ and $\xi$ is publicly available to both the investor and the two firms.

We use a three-stage model to illustrate the search process used by the investor and the pricing strategy adopted by the firm. At stage one, each firm makes an offer to the market at prices $p_i$ and $p_j$, respectively. At stage two, after observing
the IPO prices, the investor randomly selects a firm (namely, the first firm) to determine the price for the firm’s stock. Here, we assume that the investor does not make a completely uninformed investment decision but will conduct a search on at least the first target firm.9 This investigation will incur search costs $c_1$, but will refine the investor’s information about the firm and generate a post-search stock value $\hat{V}_x$ ($x = i, j$). The investor will then make a purchase decision based on this realized value $\hat{V}_x$.

The firms, on the other hand, are unable to observe the investor’s post-search value $\hat{V}_x$ but they know that $\hat{V}_x$ follows a uniform distribution $\Omega$ around the intrinsic value $V_x$ on a narrow support $[V_x - a, V_x + a]$, where $V_x$ is the mean and $a > 0$ measures the deviation of $\hat{V}_x$ from $V_x$. Since the search will refine the investor’s information, we implicitly assume $(V_x + a) - (V_x - a) < (\hat{V}_x + \xi) - (\hat{V}_x - \xi)$, that is, $a < \xi$.10 This means that before the search, the firm knows the distribution $\Omega[V_x - a, V_x + a]$ from which the investor will draw for the search, but is unable to observe the investor’s post-search value $\hat{V}_x$. Given this, a firm’s pricing decision can only be based on its estimate of the range $[V_x - a, V_x + a]$, while the investor’s purchase decision is based on the realized post-search value $\hat{V}_x$ from the first search.

After the search, the game reaches the third stage. At this stage, if we assume that the first firm being searched is $F_i$, the investor will then make a decision based on the search result $\hat{V}_i$. The investor faces three choices according to the realized $\hat{V}_i$. First, if $\hat{V}_i$ is high enough (in relation to $p_i$, $p_j$, expected $\hat{V}_j$, and future search costs), the investor will accept this firm’s offer immediately and not conduct the second search. When this happens, the game stops. Second, if the realized $\hat{V}_i$ is not high enough while the second offer $p_j$ is sufficiently attractive (in relation to $p_i$, future search costs, and expected $\hat{V}_j$), the investor will conduct a second search for the value of the second firm (thus incurring a second search cost $c_2$), and then decide to either accept $p_i$ or $p_j$, or reject both offers. If the investor decides to reject both offers, we assume that the investor will incur future search costs $c_3$ to investigate another investment opportunity.11 Third, if the realized $\hat{V}_i$ after the first search is not high enough while the second offer $p_j$ is not attractive either (in relation to $p_i$, $c_2$, $c_3$, and expected $\hat{V}_j$), the investor will reject both offers immediately after the first search. This, again, will incur future search costs $c_3$ for the investor. We assume there is a learning factor $\gamma \in [0, 1]$ such that $c_2 = \gamma c_1$ and $c_3 = \gamma^2 c_1$. If $\gamma$ is small, the information gathered in this search will be more helpful to the investor in the next search, and the search costs decrease more quickly over time.12 Given this, although the investor has an incentive to minimize the total search costs (by searching fewer firms), the impact of search costs will be reduced if there is a strong learning effect.

Both firms are aware of the investor’s strategy and the information set the investor possesses. The only thing the firms do not know (but the investor will know if when a search is conducted) is the realized value from the search, $\hat{V}_i(x = i, j)$. However, the firms know that this realized value will be drawn from a distribution $\Omega[V_x - a, V_x + a]$ and they will price their IPOs according to this distribution.
with the investor’s strategy in mind. We assume that there are costs for a firm to issue an IPO. If an IPO fails, the firm will incur deadweight costs \( V_x - W_x \) (such as the loss of reputation, a signal that the firm’s assets are not attractive, the loss of issuance expenses, and the inability to be traded in the stock markets), where \( W_x \) is the value of the firm if the issue fails.\(^{13}\) Given this, holding everything else constant, the firm has an incentive to adopt a pricing strategy that will minimize the possibility of incurring the deadweight costs.

Exhibit 1 illustrates the investor’s search strategy and the firms’ pricing decisions. In the following sub-sections, we will first model the interactions between the investor and the two firms. In other words, we will model a best-efforts contract where underwriters do not play a role. In section three, we will incorporate the underwriter into the model and explicitly discuss the contract choice between firm commitment and best efforts for the IPO firms.

**Decision Rules and Payoffs**

As shown in Exhibit 1, after the firms announce their IPO prices, there are 10 possible outcomes (which we term terminal nodes) depending on the actions the investor will take. After observing the prices, the investor will first randomly select a firm (either \( F_i \) or \( F_j \)) to investigate its stock value. If \( F_i \) is selected (with a 50% probability), there are five possible outcomes (or terminal nodes). Exhibit 2 summarizes the results.

As Exhibit 2 shows, the first terminal node is \( NA_i \). This happens when, after the first search, the investor decides to purchase the stock of \( F_i \) without conducting a second search. The payoff vector (for investor, \( F_i, F_j \)) of this node is \((V_i - p_i - c_1, p_i, V_j)\). This means that the investor pays \( p_i \) and the first search cost \( c_1 \) to get a stock with an intrinsic value \( V_i \). Firm \( F_i \) sells its stock at \( p_i \), while \( F_j \) keeps its stock. Since the IPO is not a failure and can be considered by other investors in the future, \( F_j \) retains its original intrinsic value \( V_j \).

The second terminal node is \( NO_i \). This happens when, after the first search, the investor decides to purchase the stock of \( F_j \) (and will not conduct the second search). The payoff vector of this node is \((-c_1 - c_3, W_i, V_j)\).\(^{14}\) This means that the investor pays the first search cost \( c_1 \) and the third search costs for other alternatives. Since \( F_i \) has been searched, but has not sold its stock (hence the IPO fails), the firm incurs deadweight costs \((V_i - W_i)\) with its stock value dropping to \( W_i \). Since \( F_j \) has not been searched and can be considered by other investors in the future, \( F_j \) retains its original intrinsic value \( V_j \).

The last three terminal nodes are \( SA_i, SR_i, \) and \( SO_i \). They are the three options that the investor faces when conducting a second search for the stock value of \( F_j \) (and, hence, incurring the search cost \( c_2 \)). After the second search, if the investor decides to buy the stock of \( F_j \), the game will end at terminal node \( SA_i \) with a payoff vector \((V_i - p_i - c_1 - c_2, p_i, W_j)\). This means that the investor pays \( p_i \) and
Firm $F_i$ and firm $F_j$ issue IPOs at prices $p_i$ and $p_j$, respectively. Observing the prices, the investor randomly picks a firm and searches for its intrinsic value at a cost $c_1$. The game proceeds on the left branch if $F_i$ is picked. Observing $F_i$'s post-search value, the investor decides to (1) accept $p_i$ immediately (which ends the game at $NA_i$), (2) reject both $p_i$ and $p_j$ immediately and incur costs $c_3$ to search for future investment opportunities (which ends the game at $NO_i$), or (3) search $F_j$'s value at a cost $c_2$. If the investor continues the search, the investor will decide to (1) accept $p_j$, (2) accept $p_j$, or (3) reject both (which again costs $c_3$ to search for future investment opportunities). This will end the game at $SA_i$, $SR_i$, $SO_i$, or $NO_i$. The game proceeds on the right branch if $F_j$ is picked first. When this happens, the game ends at $NA_j$, $NO_j$, $SA_j$, $SR_j$, or $SO_j$. Note that the search costs are correlated with a learning factor $\gamma \in [0, 1]$, where $c_2 = \gamma c_1$ and $c_3 = \gamma^2 c_1$. The payoffs to the relevant parties at each terminal node are described in Exhibit 2.
two search costs ($c_1$ and $c_2$) to get a stock with an intrinsic value $V_j$. Firm $F_i$ fails to sell its stock at $p_j$, incurs a deadweight loss, $V_j - W_j$, and its stock value drops to $W_j$. Firm $F_i$, on the other hand, sells its stock and receives a payoff $p_i$.

If, however, the investor decides to buy the stock of $F_j$ after the second search, the game will end at terminal node $SR_j$ with a payoff vector $(V_j - p_j - c_1 - c_2, W, p_i, p_j)$. In this case, the investor pays $p_j$ and two search costs ($c_1$ and $c_2$) to get a stock with an intrinsic value $V_j$. Firm $F_j$ fails to sell its stock at $p_i$ and incurs a deadweight loss $V_i - W_i$ (with its stock value dropping to $W_i$), while firm $F_j$ sells its stock and receives a payoff $p_j$.

Lastly, if after the second search the investor decides not to buy any stock, the game will end at terminal node $SO_j$. Under this circumstance, the payoff vector for the investor, and firms $F_i$ and $F_j$, is $(-c_1 - c_2 - c_3, W, W)$. The payoff to the investor is $c_1 - c_2 - c_3$ because the investor does not purchase any of the two stocks and will incur further search costs for her future investment. Since we assume that there are deadweight costs $V_i - W_i$ when an IPO fails, if both $F_i$ and $F_j$ fail to sell their stocks, the values of the firms’ stocks drop to $W_i$ and $W_j$, respectively.
If, at the beginning of the game, the investor randomly selects to search $F_j$ (instead of $F_i$), there will also be five terminal nodes ($NA_j$, $NO_j$, $SA_j$, $SR_j$, and $SO_j$). As Exhibit 2 shows, the payoffs of these five terminal nodes correspond to that of $NA_i$, $NO_i$, $SA_i$, $SR_i$, and $SO_i$. This is true because the decision rules are the same under both circumstances with the only difference being that $F_j$ (instead of $F_i$) is searched first. Given this, the payoffs of $NA_j$, $NO_j$, $SA_j$, $SR_j$, and $SO_j$ simply mirror the payoffs of $NA_i$, $NO_i$, $SA_i$, $SR_i$, and $SO_i$, and we only need to switch the payoffs between $F_i$ and $F_j$.

Exhibit 2 summarizes the details on the payoffs to the three players (investor, firm $F_i$, and firm $F_j$) at each of the 10 terminal nodes. After observing the prices ($p_i$ and $p_j$) offered by the two firms, the investor will make an optimal decision based on these 10 possible payoffs. In other words, given the prices offered by the firms, the magnitudes of the search costs ($c_1$, $c_2$, and $c_3$) and the deadweight costs, the investor will select one of the 10 terminal nodes to maximize the payoff. With backward induction, the firms understand the investor’s decision rule and will set a corresponding price level that maximizes their payoffs. The next two subsections discuss the maximization problems of the investor and the firms.

**The Investor’s Optimization Decision**

As assumed earlier, there is a 50% probability for each firm to be searched first. After the first search, the investor obtains a realized $\hat{V}_i$. Knowing $p_i$, $p_j$, realized $\hat{V}_i$, and the expected (not realized) value of $\hat{V}_i$, the investor chooses to either (1) immediately accept $p_i$ ($NA_i$), (2) to immediately reject both $p_i$ and $p_j$ ($NO_i$), or (3) to conduct a second search. If there is a second search, the investor will know $p_i$, $p_j$, realized $\hat{V}_i$, and realized $\hat{V}_j$ after this search. The investor will then choose to either (1) accept $p_j$ ($SA_i$), (2) accept $p_j$ ($SR_i$), or (3) reject both $p_i$ and $p_j$ ($SO_i$).

As defined earlier, before the first search the investor estimates that the stock value $V_{bi}$ follows a uniform distribution $U[\bar{V}_i - \xi, \bar{V}_i + \xi]$. We also assume that the firms estimate that the post-search perceived stock value $\hat{V}_i$ follows a uniform distribution $U[V_i - a, V_i + a]$. Consistent with the discussions earlier, we have $a < \xi$.

The investor’s optimization problem can be solved with backward deduction, starting with comparing nodes $SA_i$, $SR_i$, and $SO_i$ given that the investor proceeded with the second search, and then comparing nodes $NA_i$, $NO_i$ and the choice of conducting the second search. Lemma 1 reports the investor’s optimal decisions and Exhibit 3 summarizes the results.

**Lemma 1.** With two firms $F_i$ and $F_j$ simultaneously offering IPOs at prices $p_i$ and $p_j$, respectively, the investor’s decision rules after searching the first firm $F_i$’s stock value are:

1. If $\hat{V}_i - p_i \geq c_3$ and $\hat{V}_i - p_i \geq \bar{V}_j + \xi - p_j - 2\sqrt{c_2\xi}$, immediately purchase the stock at $p_i$;

"
### Exhibit 3 | Investor’s Decision Rules

<table>
<thead>
<tr>
<th>Route ([k])</th>
<th>Post-1st-Search Observation</th>
<th>Investor’s Decision</th>
<th>Post-2nd-Search Observation</th>
<th>Terminal Node</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1]</td>
<td>(V_i - p_i \geq -c_2) and (V_i - p_i \geq \tilde{V}_i + \xi - p_i - 2\sqrt{c_2\xi})</td>
<td>No 2nd Search</td>
<td>–</td>
<td>NA</td>
</tr>
<tr>
<td>[2]</td>
<td>(V_i - p_i \geq -c_3) and (V_i - p_i &lt; \tilde{V}_i + \xi - p_i - 2\sqrt{c_2\xi})</td>
<td>2nd Search</td>
<td>(V_i - p_i \geq \tilde{V}_i - p_i)</td>
<td>SA</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(V_i - p_i &lt; \tilde{V}_i - p_i)</td>
<td>SR</td>
</tr>
<tr>
<td>[3]</td>
<td>(V_i - p_i &lt; -c_3) and (p_i \geq \tilde{V}_i + \xi + 2\sqrt{c_2\xi} + c_3)</td>
<td>No 2nd Search</td>
<td>–</td>
<td>NO</td>
</tr>
<tr>
<td>[4]</td>
<td>(V_i - p_i &lt; -c_3) and (p_i &lt; \tilde{V}_i + \xi + 2\sqrt{c_2\xi} + c_3)</td>
<td>2nd Search</td>
<td>(V_i - p_i \geq -c_3)</td>
<td>SR</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(V_i - p_i &lt; -c_3)</td>
<td>SO</td>
</tr>
</tbody>
</table>

**Notes:** This table summarizes the investor’s decision rules. \(NA, NO, SA, SR,\) and \(SO\) are the indexes for outcomes (see Exhibit 1); \(p_i\) and \(p_j\) are the two IPO prices; \(\tilde{V}_i\) and \(\tilde{V}_j\) are the stock values perceived by the investor after her searches; \(c_2\) and \(c_3\) are the investor’s search costs for the second stock and further investment opportunities, respectively; routes [1], [2], [3], and [4] are the four routes that an investor can take after the first search: [1] accept the first offer without a second search, [2] conduct a second search and then decide to accept one offer, [3] reject both offers without a second search, or [4] conduct a second search and then decide to either accept the second offer or reject both offers.
[2] If \( \hat{V}_i - p_i \geq -c_3 \) and \( \hat{V}_i - p_i < \bar{V}_j + \xi - p_j - 2\sqrt{c_2\xi} \), conduct a second search, then purchase the stock of firm \( F_i \) at \( p_i \) if \( \hat{V}_i - p_i \geq \hat{V}_j - p_j \) or purchase the stock of firm \( F_j \) at \( p_j \) if \( \hat{V}_i - p_i < \hat{V}_j - p_j \);

[3] If \( \hat{V}_i - p_i < -c_3 \) and \( p_j \geq \bar{V}_j + \xi + 2\sqrt{c_2\xi} + c_3 \), immediately reject both offers at \( p_i \) and \( p_j \);

[4] If \( \hat{V}_i - p_i < -c_3 \) and \( p_j < \bar{V}_j + \xi + 2\sqrt{c_2\xi} + c_3 \), conduct a second search, then purchase the stock of firm \( F_j \) at \( p_j \) if \( \hat{V}_j - p_j \geq -c_3 \), or reject both offers at \( p_i \) and \( p_j \) if \( \hat{V}_j - p_j < -c_3 \).

**Proof.** See the Appendix.

Lemma 1 indicates that the investor’s first decision is to compare the magnitudes of \( \hat{V}_i - p_i \) and \(-c_3\). This is true because if the investor decides not to purchase one of the two stocks, a third search cost will be incurred, \( c_3 \), for future investment opportunities. Given this, the investor is better off by purchasing a stock (\( F_i \) or \( F_j \)) as long as the difference between the value and the price of the first searched stock \( (\hat{V}_i - p_i) \) is higher than \(-c_3\). This condition leads to routes [1] and [2] identified by Lemma 1. Under this condition (to purchase at least one stock), if the price of the other stock \( p_j \), the second search cost \( c_2 \), and the pre-search price range \( \xi \) are high enough (hence the magnitude of \( p_j + 2\sqrt{c_2\xi} \) is large enough), the investor will stop searching and purchase the first stock. This will lead to route [1] and the investor’s payoff is terminal node \( \text{NA} \) \( (\hat{V}_i - p_i - c_1) \). On the other hand, if the price of the other stock \( p_j \), the second search cost \( c_2 \), and the pre-search information uncertainty \( \xi \) are low enough (hence the magnitude of \( p_j + 2\sqrt{c_2\xi} \) is small enough), then the investor will have an incentive to conduct a second search for the value of the other stock. After the second search, the investor will purchase the stock with a higher value (that is, the larger of the difference between \( \hat{V}_i - p_i \) and \( \hat{V}_j - p_j \)). With this second search, the investor will take route [2] identified in Lemma 1, with a payoff terminal node \( \text{SA} \) or \( \text{SR} \) \( (\hat{V}_i - p_i - c_1 - c_2 \) or \( \hat{V}_j - p_j - c_1 - c_2 \)).

When the difference between the value identified by the investor and the price of the first stock searched \( (\hat{V}_i - p_i) \) is lower than \(-c_3\), the investor will immediately decide not to purchase this particular stock under any circumstance. The investor will not conduct a second search if the price of the other stock is also too high (higher than the upper boundary of the estimated intrinsic value of the stock and the third search cost, or \( p_j \geq \bar{V}_j + \xi + 2\sqrt{c_2\xi} + c_3 \)). If the investor decides not to conduct a second search at this stage, no stock will be purchased and the investor will wait for future investment opportunities. Consequently, the investor will take route [3] and end up at terminal node \( \text{NO} \) with a payoff \(-c_3 - c_1\). However, if the offering price of the other stock is not extremely high \( (p_j < \bar{V}_j + \xi + 2\sqrt{c_2\xi} + c_3) \), the investor will conduct a second search. This will lead to route [4]. If the realized value of the second firm’s stock as relative to its offering price is sufficiently high \( (\hat{V}_j - p_j \geq -c_3) \), the investor will purchase the stock of the second firm and end up at terminal node \( \text{SR} \) with a payoff \( \hat{V}_j - p_j - c_1 - c_2 \). Alternatively, if the realized value of the second firm’s stock as relative to its
offering price is sufficiently low ($\hat{V}_j - p_j < -c_3$), the investor will not purchase any stock at this stage, ending up at terminal node $SO$ with a payoff $-c_1 - c_2 - c_3$.

Routes [1], [2], [3], and [4] represent all possible moves by the investor. Clearly, the route the investor will select depends on the prices set by the firms ($p_i$ and $p_j$), the magnitudes of the search costs ($c_1$, $c_2$, $c_3$), and the pre-search firm value distribution parameters ($\hat{V}_j$, $\xi$). As Exhibit 3 indicates, the investor will go along either route [1] or route [2] if she observes $\hat{V}_i - p_i \geq -c_3$. The final decision between these two routes is determined by the magnitudes of the prices $p_i$ and $p_j$, the post-search perceived firm value $\hat{V}_i$, the second search cost $c_2$, and the upper boundary of the estimated intrinsic stock value $\bar{V}_j + \xi$. If $\hat{V}_i - p_i \geq \bar{V}_j + \xi - p_j - 2\sqrt{c_2}\xi$, the investor will pass up the second search and accept $p_i$ immediately. Note that holding $\hat{V}_i$, $p_j$, $\bar{V}_j$, and $\xi$ constant, a lower $p_i$ or a higher $c_2$ will make the “no second search” route more likely. In this sense, there is an incentive for a firm to lower its offering price to reduce the investor’s incentive to shop around.

The investor will go along either route [3] or route [4] if $\hat{V}_i - p_i < -c_3$. If the investor selects route [3] or route [4], a likely outcome is that the investor will reject both offers. Given this, the investor may have an incentive to accept a higher price if search costs are high.

The results derived in this section seem quite intuitive. If the price set by the firm is too high or if search costs are too low, the investor will be more likely to conduct another search for investment opportunities. If the investor does not purchase the firm’s stock, the firm suffers because of the deadweight costs. Given this, firms have incentives to price their stocks low enough to retain the investor. On the other hand, if the search costs are high, the investor may have an incentive to purchase the stock at a higher price to avoid the need for a further search.

**The Firm’s Optimization Strategy**

With the investor’s four routes (as summarized in Exhibit 3) in mind, a firm will select an IPO price that maximizes the expected payoffs. Since there are four possible investor decision rules, there will be four possible payoffs. Panel A of Exhibit 4 reports the payoffs of firm $F_i$ and firm $F_j$ given the route selected by the investor. (The payoffs of the firms are extracted from the last column of Exhibit 2 and are self explanatory.) To simplify our presentation, we define $A_i = V_i - a$, $B_i = p_i - c_3$, $C_i = \bar{V}_j + \xi + p_i - p_j - 2\sqrt{c_2}\xi$, $D_i = V_i - a$, and $\psi = \bar{V}_j + \xi + 2\sqrt{c_2}\xi + c_3$. It should be noted that the magnitudes of $B_i$, $C_i$, and $\psi$ decide the route the investor will take. Since the magnitudes of $B_i$ and $C_i$ are functions of $p_i$, the magnitude of $p_i$ (or the pricing strategy of the firms) will affect the magnitudes of $B_i$ and $C_i$ (which affect the route the investor will take). The route the investor takes will, in turn, decide the profits of the firm.

Panel A of Exhibit 4 shows that the payoffs of routes [1] and [3] are deterministic since there is no need to decide whether a second search is required. However,
**Exhibit 4 | Firms’ Payoffs**

Panel A: Firms’ payoffs at each terminal node

<table>
<thead>
<tr>
<th>Route [k]</th>
<th>Post-1st-Search Observation</th>
<th>Investor’s Decision</th>
<th>Post-2nd-Search Observation</th>
<th>Terminal Node</th>
<th>F’s Payoff</th>
<th>F’s Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1]</td>
<td>( \hat{V}_1 \geq \mathbf{B}, ) and ( \hat{V}_1 \geq \mathbf{C}, )</td>
<td>No 2nd Search</td>
<td>-</td>
<td>NA,</td>
<td>( p_i ),</td>
<td>( V_i )</td>
</tr>
<tr>
<td>[2]</td>
<td>( \hat{V}_1 \geq \mathbf{B}, ) and ( \hat{V}_1 &lt; \mathbf{C}, )</td>
<td>2nd Search</td>
<td>( \hat{V}_1 \leq \hat{V}_1 - p_i + p_j )</td>
<td>SA,</td>
<td>( p_i ),</td>
<td>( W_j )</td>
</tr>
<tr>
<td>[3]</td>
<td>( \hat{V}_1 &lt; \mathbf{B}, ) and ( p_i \geq \psi )</td>
<td>No 2nd Search</td>
<td>-</td>
<td>NO,</td>
<td>( W_i ),</td>
<td>( p_i )</td>
</tr>
<tr>
<td>[4]</td>
<td>( \hat{V}_1 &lt; \mathbf{B}, ) and ( p_i &lt; \psi )</td>
<td>2nd Search</td>
<td>( \hat{V}_1 \leq \hat{V}_1 - p_i - c_3 )</td>
<td>SR,</td>
<td>( W_i ),</td>
<td>( p_i )</td>
</tr>
</tbody>
</table>

Panel B: Firms’ expected payoffs when searched first (or second)

<table>
<thead>
<tr>
<th>Route [k]</th>
<th>II [1]: Firm F’s Expected Payoff if Searched First</th>
<th>II [2]: Firm F’s Expected Payoff if Searched Second</th>
<th>II [3]: Firm F’s Expected Payoff if Searched Second</th>
<th>II [4]: Firm F’s Expected Payoff if Searched Second</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1]</td>
<td>( V_i )</td>
<td>( V_i )</td>
<td>( P_i )</td>
<td>( P_i )</td>
</tr>
<tr>
<td>[2]</td>
<td>( \int_{V_i - \alpha}^{V_i + \alpha} \frac{p_i}{2a} dV_i )</td>
<td>( \int_{V_i - \alpha}^{V_i + \alpha} \frac{W_i}{2a} dV_i )</td>
<td>( \int_{V_i - \alpha}^{V_i + \alpha} \frac{p_i}{2a} dV_i )</td>
<td>( \int_{V_i - \alpha}^{V_i + \alpha} \frac{W_i}{2a} dV_i )</td>
</tr>
<tr>
<td>[3]</td>
<td>( W_i )</td>
<td>( W_i )</td>
<td>( W_i )</td>
<td>( W_i )</td>
</tr>
<tr>
<td>[4]</td>
<td>( \int_{V_i - \alpha}^{V_i + \alpha} \frac{p_i}{2a} dV_i )</td>
<td>( \int_{V_i - \alpha}^{V_i + \alpha} \frac{W_i}{2a} dV_i )</td>
<td>( \int_{V_i - \alpha}^{V_i + \alpha} \frac{p_i}{2a} dV_i )</td>
<td>( \int_{V_i - \alpha}^{V_i + \alpha} \frac{W_i}{2a} dV_i )</td>
</tr>
</tbody>
</table>

Notes: This table summarizes firms’ payoffs that correspond to the investor’s decision. Panel A shows the payoffs at each terminal node, while Panel B shows the expected payoffs on each branch along each of the four routes. NA, NO, SA, SR, SO, NA, NO, SA, SR, and SO are the indexes for outcomes (see Exhibit 1); \( p_i \) and \( p_j \) are the IPO prices; \( V_i \) and \( \hat{V}_i \) are the intrinsic stock values; \( V_i \) and \( \hat{V}_i \) are the stock values perceived by the investor after her searches; \( W_i \) and \( W_j \) are the firms’ payoffs if their IPOs cannot be sold at the initial offer prices; \( \alpha \) is the post-search information bias; \( c_1, c_2 \) and \( c_3 \) are the investor’s search costs for the first stock’s value, the second stock’s value and further investment opportunities, respectively; \( \mathbf{B}, \mathbf{C}, \) and \( \psi \) are abbreviations with \( \mathbf{B} = p_i - c_2 \), \( \mathbf{C} = V_i + \xi + p_i - p_j - 2\sqrt{c_2\xi} \), and \( \psi = V_i + \xi + 2\sqrt{c_2\xi} + c_3 \).
the outcomes of routes [2] and [4] are stochastic since their payoffs depend on whether the investor will conduct the second search. As mentioned earlier, the investor will make this decision based on the post-search firm value \( \hat{V}_i \) (which the firm is unable to observe). Consequently, the expected payoffs of routes [2] and [4] depend upon the distribution of \( \hat{V}_i \in U[V_i - a, V_i + a] \) (on which the firm has information).

In addition, since a firm does not know if firm \( F_i \) or firm \( F_j \) will be searched first by the investor, the firm will have to estimate its payoffs under both circumstances. Given this, if \( F_i \) is searched first, the second column of Panel B reports the expected payoffs of each route of \( F_i \) [defined as \( E(\Pi_{IL}^k) \), where \( k \in (1,2,3,4) \)]. If \( F_j \) is searched first, the fifth column of the same panel reports the expected payoffs of each route of \( F_j \) [defined as \( E(\Pi_{IJ}^k) \), where \( k \in (1,2,3,4) \)]. If \( F_i \) (or \( F_j \)) is searched second, the third (or the fourth) columns of the same panel reports the expected payoffs of each route of \( F_i \) [defined as \( E(\Pi_{IL}^k) \), where \( k \in (1,2,3,4) \)] or \( F_j \) [defined as \( E(\Pi_{IJ}^k) \), where \( k \in (1,2,3,4) \)]. The detailed expected payoff of each route for each firm is reported in Panel B of the same table.

We can use \( E(\Pi_{IL}^2) \) to illustrate the concept. It is clear from the third row of column 4 in Panel A of Exhibit 4 that the firm will conduct a second search if \( \hat{V}_i \geq B \) and \( \hat{V}_i < C \). If the realized value is \( \hat{V}_j \leq \hat{V}_i - p_i + p_j \), then the firm’s payoff is \( p_i \). If the realized value \( \hat{V}_j > \hat{V}_i - p_i + p_j \), then the firm’s payoff is \( W_i \). Given these, the expected payoff for \( F_i \) (when the investor takes route [2] and when the firm is searched first by the investor) is \( \Pi_{IL}^{[2]} = \int_{\hat{V}_i - p_i + p_j}^{\hat{V}_i + a} p_i/2a \ d\hat{V}_j + \int_{\hat{V}_i - p_i + p_j}^{\hat{V}_i + a} W_i/2a \ d\hat{V}_j \), as reported in the third row of column 2 in Panel B of Exhibit 4.

From Panels A and B of Exhibit 4, it is clear that a firm’s payoff depends on the route selected by the investor or on whether the firm was searched first or not. Since there are many possible routes and the pricing decisions of the firms affect the route, it is not feasible for us to find a global optimal pricing decision by examining all routes in one equation. To handle this issue, we follow the methodology employed by Lee (1994). In Lee’s paper, a seller’s expected payoff function depends on the possible orders of a buyer’s decision boundaries. To maximize the seller’s payoff, Lee solves for the optimal price that maximizes a seller’s expected payoff function that matches each possible boundary order and other necessary conditions. Lee then discusses the feasibility of the optimal price based on its consistency with all the conditions that are used to derive the optimal prices.

To follow this methodology, we first need to establish all the possible boundary orders. We know that, when \( F_i \) is searched first, the realized value \( \hat{V}_i \) can only fall into a range with boundaries that may take four possible values: \( V_i - a \) (defined as \( A_i \) before), \( p_i - c_3 \) (defined as \( B_i \) before), \( \hat{V}_j + \xi + p_i - p_j - 2\sqrt{c_2}\xi \) (defined as \( C_i \) before), and \( V_i + a \) (defined as \( D_i \) before). Among these four variables, we know that \( A_i = V_i - a \) is the lower bound of \( p_i \) and \( D_i = V_i + a \) is the upper bound of \( p_i \) because the firm will not offer a price lower than the lowest value an
With the constraint that \( A_i < D_i \), the sequence of the four possible values (\( A_i \), \( B_i \), \( C_i \), and \( D_i \)) can take a total of \( \frac{1}{2} p_i^4 = 12 \) possible orders. Combining with another constraint associated with \( p_j > \psi = \bar{V}_j + \xi + 2\sqrt{c_2\xi} + c_3 \), there are a total of \( 12 \times 2 = 24 \) possible situations where firm \( F_i \)'s expected payoff function can take different formats.\(^{16} \)

Exhibit 5 provides details of the order conditions and payoffs of these 24 situations. Column 2 of the table reports the order constraint imposed by \( A_i \), \( B_i \), \( C_i \), and \( D_i \); column 3 reports the second constraint (on \( \psi \)) on the situations; and column 4 reports the implied conditions derived from the order specified in columns 2 and 3. Column 5 reports the possible routes this particular situation might reach. The last two columns report the expected payoffs of a firm if it is searched first (column 6) and if it is not (column 7).

We can use the third row (situation 2) of Exhibit 5 as an example. The third row reports an order that is bounded by \( B_i \) \( \leq A_i \) \( \leq C_i \) \( \leq D_i \) (column 2) and has a second constraint \( p_j < \psi \) (column 3). Column 4 indicates that when \( B_i \) \( \leq A_i \) \( \leq C_i \) \( \leq D_i \) holds, it follows that \( B_i \) \( \leq A_i \leq \bar{V}_i \). This is true because \( A_i \) is the lower bound of \( \bar{V}_i \). Since \( B_i \) \( \leq A_i \leq \bar{V}_i \), an examination of Panel A of Exhibit 4 indicates that the only possible routes this order can reach are routes \([\text{1}]\) and \([\text{2}]\) (as reported in column 6 of Exhibit 5). From Panel B of Exhibit 4, we know that if firm \( F_i \) is searched first, its payoff is either \( \Pi_{1}\|_{LR} \) (route \([\text{1}]\)) or \( \Pi_{2}\|_{LR} \) (route \([\text{2}]\)). In this case, the probability for the investor to take route \([\text{1}]\) and route \([\text{2}]\) are \( \int_{K_i}^{C_i} 1/2a \ d\bar{V}_i \) and \( \int_{K_i}^{C_i} 1/2a \ d\bar{V}_i \), respectively. Consequently, the expected value of being searched first for situation 2 is \( E(\Pi_{LR}) = \int_{K_i}^{C_i} \Pi_{1}\|_{LR} 1/2a \ d\bar{V}_i + \int_{K_i}^{C_i} \Pi_{2}\|_{LR} 1/2a \ d\bar{V}_i \) (as reported in column 6 of Exhibit 5). Alternatively, if the firm is not searched first, we know from Panel B of Exhibit 4 that its payoff is \( \Pi_{1}\|_{LR} \) (route \([\text{1}]\)) or \( \Pi_{2}\|_{LR} \) (route \([\text{2}]\)). In this case, the probability for the investor to take route \([\text{1}]\) and route \([\text{2}]\) are \( \int_{K_i}^{C_i} 1/2a \ d\bar{V}_j \) and \( \int_{K_i}^{C_i} 1/2a \ d\bar{V}_j \), respectively. Consequently, the expected value when a firm is searched for situation 2 is \( E(\Pi_{LR}) = \int_{K_i}^{C_i} \Pi_{1}\|_{LR} 1/2a \ d\bar{V}_j + \int_{K_i}^{C_i} \Pi_{2}\|_{LR} 1/2a \ d\bar{V}_j \) (as reported in column 7 of Exhibit 5). From columns 6 and 7, we can calculate the expected payoff of a firm for each situation as \( E(\Pi_i) = \frac{1}{2} E(\Pi_{LR}) + \frac{1}{2} E(\Pi_{LR}) \).

For each possible situation, we will first examine if the situation actually exists. This is necessary because there are multiple constraints for each situation and those constraints might conflict with each other. Nine situations (situations 1, 3, 5, 9, 11, 13, 17, 19, and 21) actually do not exist. We can use situation 1 as an example. Situation 1 requires both \( B_i \leq D_i \) and \( p_j \geq \psi \) to hold simultaneously. A detailed analysis in the Appendix proves that the two constraints cannot exist simultaneously. After taking out the nine situations that do not exist, we still have 15 feasible situations. For these 15 situations, we solve for the optimal price of each situation. This is done by solving for the offering price that maximizes the firm’s expected payoff, or:
**Exhibit 5 | 24 Situations a Firm Can Face**

<table>
<thead>
<tr>
<th>Situation Index</th>
<th>Possible Order (from Low to High)</th>
<th>Possible Condition of $p_j$</th>
<th>Satisfied Condition</th>
<th>Possible Routes</th>
<th>$E(I)_{L}$</th>
<th>$E(I)_{R}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$B_i, A_i, C_i, D_i$</td>
<td>$p_i \geq \psi$</td>
<td>$B_i \leq A_i \leq \hat{V}_i$</td>
<td>none</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>2</td>
<td>$B_i, A_i, C_i, D_i$</td>
<td>$p_i &lt; \psi$</td>
<td>Same as above</td>
<td>$[1], [2]$</td>
<td>$\int_{c}^{d} II_{i}^{[1]} \frac{1}{2a} d\hat{V}_i$</td>
<td>$\int_{c}^{d} II_{i}^{[1]} \frac{1}{2a} d\hat{V}_i$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$+ \int_{a}^{c} II_{i}^{[2]} \frac{1}{2a} d\hat{V}_i$</td>
<td>$+ \int_{a}^{c} II_{i}^{[2]} \frac{1}{2a} d\hat{V}_i$</td>
</tr>
<tr>
<td>3</td>
<td>$A_i, B_i, C_i, D_i$</td>
<td>$p_i \geq \psi$</td>
<td>$A_i \leq D_i \leq \hat{V}_i$</td>
<td>none</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>4</td>
<td>$A_i, B_i, C_i, D_i$</td>
<td>$p_i &lt; \psi$</td>
<td>Same as above</td>
<td>$[1], [2], [4]$</td>
<td>$\int_{a}^{b} II_{i}^{[1]} \frac{1}{2a} d\hat{V}_i$</td>
<td>$\int_{a}^{b} II_{i}^{[1]} \frac{1}{2a} d\hat{V}_i$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$+ \int_{c}^{b} II_{i}^{[2]} \frac{1}{2a} d\hat{V}_i$</td>
<td>$+ \int_{c}^{b} II_{i}^{[2]} \frac{1}{2a} d\hat{V}_i$</td>
</tr>
<tr>
<td>5</td>
<td>$A_i, C_i, B_i, D_i$</td>
<td>$p_i \geq \psi$</td>
<td>if $B_i \leq \hat{V}_i$ then $C_i \leq \hat{V}_i$</td>
<td>none</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>6</td>
<td>$A_i, C_i, B_i, D_i$</td>
<td>$p_i &lt; \psi$</td>
<td>Same as above</td>
<td>$[1], [4]$</td>
<td>$\int_{b}^{a} II_{i}^{[1]} \frac{1}{2a} d\hat{V}_i$</td>
<td>$\int_{b}^{a} II_{i}^{[1]} \frac{1}{2a} d\hat{V}_i$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$+ \int_{a}^{c} II_{i}^{[2]} \frac{1}{2a} d\hat{V}_i$</td>
<td>$+ \int_{a}^{c} II_{i}^{[2]} \frac{1}{2a} d\hat{V}_i$</td>
</tr>
<tr>
<td>7</td>
<td>$A_i, C_i, D_i, B_i$</td>
<td>$p_i \geq \psi$</td>
<td>$B_i \geq D_i \geq \hat{V}_i$</td>
<td>$[3]$</td>
<td>$W_i$</td>
<td>$V_i$</td>
</tr>
<tr>
<td>8</td>
<td>$A_i, C_i, D_i, B_i$</td>
<td>$p_i &lt; \psi$</td>
<td>Same as above</td>
<td>$[4]$</td>
<td>$W_i$</td>
<td>$W_i$</td>
</tr>
<tr>
<td>9</td>
<td>$B_i, C_i, A_i, D_i$</td>
<td>$p_i \geq \psi$</td>
<td>$B_i \leq A_i \leq \hat{V}_i$</td>
<td>none</td>
<td>$-$</td>
<td>$-$</td>
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</tbody>
</table>
### Exhibit 5 | (continued)

#### 24 Situations a Firm Can Face

<table>
<thead>
<tr>
<th>Situation Index</th>
<th>Possible Order (from Low to High)</th>
<th>Possible Condition of $p_i$</th>
<th>Satisfied Condition</th>
<th>Possible Routes</th>
<th>$E(II)_l$</th>
<th>$E(II)_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>$B_i, C_i, A_i, D_i$</td>
<td>$p_i &lt; \psi$</td>
<td>Same as above</td>
<td>[1] $p_i$</td>
<td>$V_i$</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>$C_i, B_i, A_i, D_i$</td>
<td>$p_i \geq \psi$</td>
<td>Same as above</td>
<td>none</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>12</td>
<td>$C_i, B_i, A_i, D_i$</td>
<td>$p_i &lt; \psi$</td>
<td>Same as above</td>
<td>[1] $p_i$</td>
<td>$V_i$</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>$C_i, A_i, B_i, D_i$</td>
<td>$p_i \geq \psi$</td>
<td>none</td>
<td></td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>14</td>
<td>$C_i, A_i, B_i, D_i$</td>
<td>$p_i &lt; \psi$</td>
<td>[1], [4]</td>
<td>$\int_{B_i}^{D_i} \frac{1}{2a} d\tilde{V}_i$</td>
<td>$\int_{B_i}^{D_i} \frac{1}{2a} d\tilde{V}_i$</td>
<td>$\int_{A_i}^{B_i} \frac{1}{2a} d\tilde{V}_i$</td>
</tr>
<tr>
<td>15</td>
<td>$C_i, A_i, D_i, B_i$</td>
<td>$p_i \geq \psi$</td>
<td>$B_i, D_i \geq \hat{V}_i$</td>
<td>[3] $W_i$</td>
<td>$V_i$</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>$C_i, A_i, D_i, B_i$</td>
<td>$p_i &lt; \psi$</td>
<td>Same as above</td>
<td>[4] $W_i$</td>
<td>$V_i$</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>$B_i, A_i, D_i, C_i$</td>
<td>$p_i \geq \psi$</td>
<td>$B_i \leq A_i \leq \hat{V}_i$</td>
<td>none</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>18</td>
<td>$B_i, A_i, D_i, C_i$</td>
<td>$p_i &lt; \psi$</td>
<td>Same as above</td>
<td>[2] Same as above</td>
<td>Same as above</td>
<td>Same as above</td>
</tr>
<tr>
<td>19</td>
<td>$A_i, B_i, D_i, C_i$</td>
<td>$p_i \geq \psi$</td>
<td>none</td>
<td></td>
<td>$-$</td>
<td>$-$</td>
</tr>
</tbody>
</table>
**Exhibit 5 | (continued)**

24 Situations a Firm Can Face

<table>
<thead>
<tr>
<th>Situation Index</th>
<th>Possible Order (from Low to High)</th>
<th>Possible Condition of ( p_j )</th>
<th>Satisfied Condition</th>
<th>Possible Routes</th>
<th>( E(II)_L )</th>
<th>( E(II)_R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>( A, B, D, C )</td>
<td>( p_j &lt; \psi )</td>
<td></td>
<td>[2], [4]</td>
<td>( \int_{A}^{B} I_i^{(2)} \frac{1}{2a} d\hat{V}<em>j ) + ( \int</em>{A}^{B} I_i^{(4)} \frac{1}{2a} d\hat{V}_j )</td>
<td>( \int_{A}^{B} I_i^{(2)} \frac{1}{2a} d\hat{V}<em>j ) + ( \int</em>{A}^{B} I_i^{(4)} \frac{1}{2a} d\hat{V}_j )</td>
</tr>
<tr>
<td>21</td>
<td>( A, D, B, C )</td>
<td>( p_j \geq \psi )</td>
<td>( B_j &gt; D_j \geq \hat{V}_j )</td>
<td>none</td>
<td>( W_j )</td>
<td>( W_j )</td>
</tr>
<tr>
<td>22</td>
<td>( A, D, B, C )</td>
<td>( p_j &lt; \psi )</td>
<td>Same as above</td>
<td>[4]</td>
<td>( W_j )</td>
<td>( W_j )</td>
</tr>
<tr>
<td>23</td>
<td>( A, D, C, B )</td>
<td>( p_j \geq \psi )</td>
<td>Same as above</td>
<td>[3]</td>
<td>( W_j )</td>
<td>( V_j )</td>
</tr>
<tr>
<td>24</td>
<td>( A, D, C, B )</td>
<td>( p_j &lt; \psi )</td>
<td>Same as above</td>
<td>[4]</td>
<td>( W_j )</td>
<td>( W_j )</td>
</tr>
</tbody>
</table>

Notes: This table summarizes 24 situations, in each of which a firm’s expected payoff function may take a different form. \( A, B, C, D, \) and \( \psi \) are abbreviations with \( A = V_i - a = \text{Min}(V_j) \), \( B = p_i - c_3 \), \( C = \hat{V}_j + \xi + p_i - 2V_c + c_2 \), \( D = V_i + a = \text{Max}(V_j) \), and \( \psi = \hat{V}_j + \xi + 2V_c + c_3 \) (note that \( A < D \)), where \( p_i \) and \( p_j \) are the IPO prices; \( V_i \) and \( V_j \) are the intrinsic stock values; \( \hat{V}_j \) and \( \hat{V}_j \) are the stock values after the searches; \( a \) is the post-search information bias; \( c_2 \) and \( c_3 \) are the investor's search costs for the second stock’s value and further investment opportunities, respectively; [1], [2], [3], and [4] are the four routes that the investor can take after the first search: [1] accept the first offer without a second search, [2] conduct a second search then decide to accept either the first offer or the second offer, [3] reject both offers without a second search, or [4] conduct a second search then decide to either accept the second offer or reject both offers. The notations \( I_i^{(k)} \) and \( I_i^{(m)} \) (where \( k = 1, 2, 3, 4 \)) are defined in Panel B of Exhibit 4.
Max \( E(\Pi_i) = \frac{1}{2} E(\Pi_i)_L + \frac{1}{2} E(\Pi_i)_R \) \quad (1)

where \( E(\Pi_i)_L \) is the expected payoff of \( F_i \) if \( F_i \) is searched first by the investor, and \( E(\Pi_i)_R \) is the expected payoff of \( F_i \) if \( F_j \) is searched first by the investor.

We can categorize the optimal prices for the remaining 15 situations into three categories. The first category includes the cases where we obtain an interior solution for the optimal price. These include situations 4, 14, and 20. The second category includes the cases where we obtain a corner solution. These include situations 2, 6, 10, 12, and 18. These five situations with a corner optimal solution are adjoining the situations with an interior optimal solution. A further analysis indicates that all the situations with a corner solution are dominated by the situations with an interior solution. The last group includes the cases where the optimal pricing strategy of the firm leads to a net deadweight loss. Situations 8, 16, 22, 24, 7, 15, and 23 belong to this category. Given the fact that the expected payoff is negative, a rational firm will not adopt a pricing strategy that will result in such a situation. Consequently, those situations will not exist under the optimal conditions. We, therefore, do not need to worry about them. Exhibit 6 summarizes the outcomes of all these 24 situations.

Therefore, the optimal solutions of the three situations with an interior solution will be from a firm’s optimal pricing strategy. Proposition 1 summarizes a firm’s pricing strategy. The pricing strategy will be reported as the difference between the intrinsic stock value, \( V_i \), and the optimal stock offering price set by the firm, \( p^*_i \), or

\[
D^*_i = V_i - p^*_i. 
\]  

(2)

We term \( D^*_i \) as the relative price. A positive \( D^*_i \) indicates that the firm is willing to price the stock below the intrinsic value it identifies. A negative \( D^*_i \) indicates that the offering price of the stock is higher than the firm’s intrinsic stock value.

**Proposition 1.** When firm \( F_i \) observes \( V_i - a \leq \overline{V}_j + \xi - 2\sqrt{c_2}\xi \leq V_i + a \) (case 1), its optimal pricing strategy will result in:

\[
D^*_i = V_i - p^*_i = 3a - c_3 - \sqrt{2a(6a - V_i - 2c_3 + W_i)}. 
\]  

(3)

When firm \( F_i \) observes \( V_i - a > \overline{V}_j + \xi - 2\sqrt{c_2}\xi \) (case 2), its optimal pricing strategy will result in:
Exhibit 6 | Firms’ Optimization Solutions

<table>
<thead>
<tr>
<th>Situation Index</th>
<th>Solution</th>
<th>Situation Index</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Conflicting constraints</td>
<td>13</td>
<td>Conflicting constraints</td>
</tr>
<tr>
<td>2</td>
<td>Corner, dominated by interior in situation 4</td>
<td>14</td>
<td>Interior</td>
</tr>
<tr>
<td>3</td>
<td>Conflicting constraints</td>
<td>15</td>
<td>Deadweight loss</td>
</tr>
<tr>
<td>4</td>
<td>Interior</td>
<td>16</td>
<td>Deadweight loss</td>
</tr>
<tr>
<td>5</td>
<td>Conflicting constraints</td>
<td>17</td>
<td>Conflicting constraints</td>
</tr>
<tr>
<td>6</td>
<td>Corner, dominated by interior in situation 4</td>
<td>18</td>
<td>Corner, dominated by interior in situation 20</td>
</tr>
<tr>
<td>7</td>
<td>Deadweight loss</td>
<td>19</td>
<td>Conflicting constraints</td>
</tr>
<tr>
<td>8</td>
<td>Deadweight loss</td>
<td>20</td>
<td>Interior</td>
</tr>
<tr>
<td>9</td>
<td>Conflicting constraints</td>
<td>21</td>
<td>Conflicting constraints</td>
</tr>
<tr>
<td>10</td>
<td>Corner, dominated by interior in situation 14</td>
<td>22</td>
<td>Deadweight loss</td>
</tr>
<tr>
<td>11</td>
<td>Conflicting constraints</td>
<td>23</td>
<td>Deadweight loss</td>
</tr>
<tr>
<td>12</td>
<td>Corner, dominated by interior in situations 14</td>
<td>24</td>
<td>Deadweight loss</td>
</tr>
</tbody>
</table>

Note: This table summarizes the firm’s optimization solution in each of the 24 situations, where the conditions for each situation are as described in Exhibit 5.

\[
D_i^* = V_i - p_i^* = \frac{1}{2} (V_i - a - c_3 - W_i). \tag{4}
\]

When firm \( F_i \) observes \( \bar{V_j} + \xi - 2\sqrt{c_2} \xi > V_i + a \) (case 3), its optimal pricing strategy will result in:

\[
D_i^* = V_i - p_i^* = 3a - c_3 - 2\sqrt{a}(3a - V_i - c_3 + W_i). \tag{5}
\]

All the equilibria suggest that the IPO offering price increases in the investor’s additional search costs if the investor decides not to buy one of the two stocks, increases in the estimated range of the postsearch value, and decreases in the costs if an IPO fails, or
The signs take opposite directions when we analyze the difference between the firm’s intrinsic value and the offering price \( (D_i^* = V_i - p_i^*) \), or

\[
\frac{\partial D_i^*}{\partial c_3} < 0 \quad \text{(implies} \quad \frac{\partial D_i^*}{\partial c_1} < 0 \quad \text{and} \quad \frac{\partial D_i^*}{\partial \gamma} < 0, \quad \frac{\partial D_i^*}{\partial a} < 0 \quad \text{and} \quad \frac{\partial D_i^*}{\partial W_i} < 0.
\]

\[(7)\]

**Proof.** See the Appendix.

Proposition 1 reports that the 24 situations can be re-categorized into three segments and there is a global optimal price strategy within each segment. The first segment (case 1) is set by \( V_i - a \leq \bar{V}_j + \xi - 2\sqrt{c_2\xi} \leq V_i + a \), which includes situations 1 to 8 reported in Exhibit 5. Proposition 1 reports that the global optimum in this segment is an interior solution (situation 4) that dominates two other corner solutions (situations 2 and 6) obtained in the same segment, or \( D_i^* = V_i - p_i^* = 3a - c_3 - \sqrt{2a(6a - V_i - 2c_3 + W_i)}. \) To be located in this segment (case 1), the magnitude of the search cost must be moderate (or cannot be too large or too small) on the full range of the cost spectrum so that it can satisfy the constraint set by \( V_i - a \leq \bar{V}_j + \xi - 2\sqrt{c_2\xi} \leq V_i + a. \) Therefore, we can refer to case 1 as a moderate search cost case.

The second segment (case 2) is set by \( V_i - a > \bar{V}_j + \xi - 2\sqrt{c_2\xi}, \) which includes situations 9 to 16. Proposition 1 reports that the global optimum in this segment is an interior solution (situation 14) that dominates two other corner solutions (situations 10 and 12) obtained in the same segment, or \( D_i^* = V_i - p_i^* = \frac{1}{2} (V_i - a - c_3 - W_i). \) To be located in this segment (case 2), the magnitude of the search cost must be large (or must be on the larger side of the full range of the cost spectrum) so that it can satisfy the constraint set by \( V_i - a > \bar{V}_j + \xi - 2\sqrt{c_2\xi}. \) Given this, we can refer to case 2 as an expensive search case.

The third range (case 3) is set by \( \bar{V}_j + \xi - 2\sqrt{c_2\xi} > V_i + a, \) which includes situations 17 to 24. The global optimum in this segment is an interior solution (situation 20) that dominates the corner solution (situation 18) obtained in this range, or \( D_i^* = V_i - p_i^* = \frac{1}{2} (V_i - a - c_3 - W_i). \) To be located in this segment
(case 3), the magnitude of the search cost must be on the smaller side of the full range of the cost spectrum so that it can satisfy the constraint set by $\bar{V}_j + \xi - 2\sqrt{c_2\xi} > V_i + a$. Thus, we can refer to case 3 as an inexpensive search case.

It should be noted that when the three cases are combined, the range of $c_3$ covers all possible ranges spanned by the parameters. Given this, Proposition 1 provides a complete solution set for a firm’s optimal pricing strategy. Proposition 1 also reports that the sign of the relative price $D_i^*$ can be positive or negative depending on the values of the parameters. Those variables include the deadweight loss (if an IPO fails) $W_i$, the search cost $c_1$, the speed that the investor can learn from previous searches $\gamma$, and the magnitude of the price range $a$, from which an investor will draw the firm’s value. Given these and depending on the parameter values, our model can predict both overpricing and underpricing strategies (in relation to the intrinsic value, not to the market clearance price on the first public trading day). Clearly, our model may be able to shed some light on the empirical observations that underpricing levels vary by industries and by time periods.

**Extreme Conditions**

Proposition 1 states that the optimal price level $p_i^*$ and the corresponding relative level $D_i^*$ (which is the difference between the intrinsic value the firm believes and the optimal price the firm will offer) will be affected by three factors (namely, search costs, the range of the post-search value, and the deadweight cost if an IPO fails). We will examine several extreme conditions to bring out the impact each factor might have on the price decision of the firm. Those results will then be used to explain some empirical irregularities observed in the field.

*When $W_i = V_i$.* If we assume that the downside loss is zero such that $W_i = V_i$, then Equations (3), (4), and (5) can be reduced to:

$$D_i^* = (\sqrt{3}a - c_3)(\sqrt{3}a - c_3 - \sqrt{4a}) < 0,$$  \hspace{1cm} (8)

$$D_i^* = \frac{1}{2} (-a - c_3) < 0, \text{ and}$$  \hspace{1cm} (9)

$$D_i^* = (\sqrt{3}a - c_3)(\sqrt{3}a - c_3 - \sqrt{4a}) < 0, \text{ respectively.}$$  \hspace{1cm} (10)

In this case, firms will set the price higher than their perceived intrinsic value of the firm. In other words, firms will overprice their IPOs if there is no deadweight cost when the IPOs fail. This is true because there is no penalty for the firm to set the price at a high level; however, there is a cost for the investor if the stock is not purchased. It is also clear from the equations above that the larger the $a$ or $c_3$, the higher the price the firms will offer.
When \( W_i = V_i \) and \( a = 0 \). If we assume that \( W_i = V_i \) and if the investor can come up with only one consistent estimate of the stock value (or \( a = 0 \)), then Equations (3), (4), and (5) can be reduced to \( D_i^* = -c_3 \), \( D_i^* = -\frac{1}{2} c_3 \), and \( D_i^* = -c_3 \), respectively. Under this circumstance, firms will overprice their stocks. The magnitude of overpricing depends only on the magnitude of the search costs. This makes sense because a firm knows that the investor will accept the offer price if the difference between the offering price and the investor’s perceived value of the firm’s stock is less than (or equal to) the cost of not buying the stock.

When \( W_i = V_i \) and \( c_3 = 0 \). If we assume that \( W_i = V_i \) and the search cost \( c_3 = 0 \), then Equations (3), (4) and (5) can be reduced to:

\[
D_i^* = 3a - \sqrt{12a} < 0, \quad (11)
\]
\[
D_i^* = -\frac{1}{2} a < 0, \quad (12)
\]
\[
D_i^* = 3a - \sqrt{12a} < 0, \quad \text{respectively.} \quad (13)
\]

Under this extreme circumstance, firms will definitely overprice their stocks to maximize their profits. The magnitude of the overpricing depends on the range from which the investor draws the value of the stock. The larger the range, the higher the price will be. This makes sense. If the firm knows that the estimated value the investor will draw after the search is from a range \([X - 10, X + 10]\), it is more likely to offer a price higher than that it would offer if it knows that the estimated value is from a range \([X - 1, X + 1]\).

This result is consistent with the prediction of the price-optimism type of models pioneered by Miller (1977). The price-optimism models predict that the bigger the disagreement about the value of a stock, the higher the market price relative to the true value of the stock (and, therefore, the lower the future returns of the stock). Diether, Malloy, and Scherbina (2002) provide empirical evidence to support this prediction. They find that stocks with higher dispersion in analysts’ forecasts earn significantly lower future returns. They also reject the notion that dispersion in forecasts can be viewed as a proxy for risk. Chemmanur and Paeglis (2005) examine the relationship between the quality of a firm’s management and the post-IPO performance. They find that if higher management quality is associated with lower heterogeneity in investor valuations, firms with better managers will have greater long-term stock returns. Our model predicts that issuers will offer a lower price when they see a lower heterogeneity in investor valuations (and therefore, a higher future stock return). This prediction is consistent with the result reported by Chemmanur and Paeglis (2005).

It is also quite possible that the magnitude of \( a \) might not be small. Using the product market as an example, we frequently find that the values different...
appraisers place on the same property at the same time can differ by more than 10% even though all appraisers are adequately trained and use similar (if not identical) market information to derive the property value. Similarly, sealed bids on corporate assets (during corporate takeover battles) can vary significantly among bidders even though all bidders are sophisticated players in the game. The only time that the magnitude of $a$ could be small is when the value of the firm is difficult to assess because information about its assets is difficult to obtain (or its business is too complex for the players to learn or understand). Under this circumstance, since investors cannot perform their own analyses, their estimate might depend on what is available in the market (such as the valuation provided by some analysts).

When $W_i = V_i$, $a = 0$, and $c_3 = 0$. If we further assume that $W_i = V_i$, $a = 0$, and the search cost $c_3 = 0$, then Equations (3), (4), and (5) are all reduced to $D^*_i = 0$. Under this circumstance, there is no under- or over-pricing. Indeed, firms will just price their IPOs at their perceived intrinsic values.

**Implications**

As indicated by Proposition 1, issuing firms are willing to reduce their offering prices if the cost of a failed IPO (or $W$) is high. This is true because a lower (higher) offering price will increase (decrease) the likelihood of a successful offering. It is reasonable for a firm to offer a low (high) price when the deadweight cost is high (low). Busaba, Benveniste, and Guo (2001) show that underwriters tend to reduce the underpricing level if the issuer has a credible option to withdraw the offer. This evidence seems to support our model predictions.

Given this, the asset redeployment ability of a firm could affect its pricing decision. For a firm with mainly redeployable (or tangible) assets, it will have less incentive to reduce its offering price. This is true because the firm may be able to sell its assets in the product market or to other firms at a price similar to its IPO price should the IPO fail. On the other hand, a firm with significant intangible assets (or one that believes it can sell its assets at a much higher price in the public stock market than in the product market) will want to set its IPO at a low price to avoid a failure.

This story seems very suitable for explaining the anomalous evidence on the initial day return patterns of REIT IPOs during the 1971–2000 period. Wang, Chan, and Gau (1992) report that the 87 REIT IPOs issued during the 1971–1988 period were significantly overpriced, with an average initial day return of −2.82%. However, the pricing pattern changed after 1990. Chan, Erickson, and Wang (2002) report that the 159 REIT IPOs during the 1990–2000 period are significantly underpriced, with a positive average initial day return of 2.36%. It should be noted that after 1990, REITs changed their structure from a traditional fund-like trust to one that includes an operational component in its daily routine. In other words, on top of the assets REITs own, the management component also became part of the value of REITs after 1990.17
It is fair to argue that when REITs have a fund-like structure, the deadweight cost for a failed IPO is quite small. All a REIT needs to do when an IPO fails is to either sell the individual properties it owns in the property market or sell the properties (as a package) to other large real estate investors (including other publicly-traded REITs). Given this, unless the REIT believes that it can sell its assets at a much higher price in the open stock market than in the property market, it can be argued that the cost of a failed REIT IPO is quite small. Under this scenario, our discussions above indicate that REIT IPOs will be overpriced. That is, a firm will price its IPO above its perceived intrinsic value. Equations (3), (4), and (5) indicate that if we set $a = 0.05V_i$ and $c_3 = 0.01V_i$, the initial day return is about $-3\%$ if $V_i - W_i$ is also small. This $3\%$ overpricing result is similar to the $-2.82\%$ initial day return reported for REIT IPOs during the 1971–1988 period (when REITs were still holding their traditional fund-like structure). During the 1990s, REITs changed and behaved more like operating companies. With the change, the firm value now includes intangible assets and some assets that cannot be sold in the property market. Given this, the deadweight cost of the new REITs (with operational components) should be higher than that of the old REITs (with fund-like structures). If we let $V_i - W_i = 0.10V_i$, Equations (3), (4), and (5) indicate that the underpricing level should be around $2\%$, which is consistent with the empirical result reported by Chan, Erickson, and Wang (2002) for $2.36\%$ initial day return for REIT IPOs issued during the 1990–2000 period.

Peavy (1990) reports that the 38 mutual fund IPOs issued during the 1986–1987 period are not over- or under-priced (with an average initial day return $-0.62\%$). The implications of our model are most suitable to explain this nearly $0\%$ initial day return for mutual fund IPOs. There is no doubt that most (if not all) assets of a mutual fund can be sold in the open stock market at a known price. Since price information is readily available, there is no need for an investor to incur additional costs. Given this, it is reasonable to argue that for mutual fund IPOs, the deadweight costs, search costs, and estimated range of post-search values are all zero ($V_i - W_i = 0, a = 0$, and $c_3 = 0$). Our analyses above indicate that mutual fund IPOs should be neither under-priced nor over-priced.

Michaely and Shaw (1994) report an insignificant $-0.04\%$ initial day return for the 39 MLP IPOs issued during the 1984–1988 period. They argue that this should be the case because there are no informed investors (institutional investors) in the MLP IPO market and underwriters do not need to underprice the IPOs to compensate the uninformed investors. While this explanation makes some sense, it cannot explain why REIT IPOs (that have about $10\%$ institutional participation during the 1980–1990 period) are significantly overpriced during the similar period. Again, similar to REITs, MLPs normally also hold tangible assets (such as hotels, retails, oil, and gas) in their portfolios. If a MLP disbands, it can also sell its assets in the open product markets quite easily. If this happens, the largest cost that cannot be recovered might be the partnership set-up costs. More importantly, since partnerships normally have buy-out clauses to increase the liquidity of the units they issue, it is not very important for them to have access
to the open stock market. Given this, the cost of a failed IPO should not be too great for MLP IPOs.

However, unlike REITs, the value of partnership arrangements of each individual MLP might be difficult to judge. This is the case because a MLP typically has complicated arrangements (i.e., for tax purposes) among different parties that might affect the value of the underlying assets it owns. However, there is typically not enough information for an investor to adequately assess the value of those arrangements. Given this, it might be prudent to argue that the estimated range of the post-search values should be wider for a MLP than for a modern REIT (with an operation component). Consequently, with a deadweight cost level similar to that of a modern REIT, the result of no under- or over- pricing of a MLP IPO is possible because of an offsetting effect resulting from the positive price effect of the search cost and the estimated range of post-search values and the negative price effect of the deadweight costs if the IPO fails.

Our model might also have some implications as to why the aggregate level of underpricing changes over time and differs among industries. At this moment since we do not model investor sentiment in the aftermarket, we implicitly assume that the expected initial day price trading in the open stock market should be about the same as the firm’s perceived intrinsic value of its stock. If we relax this implicit assumption, our model might shed some light on the high initial day returns observed in the hot IPO markets.

In a hot IPO market, it is possible that the expected stock price at the initial trading day is higher than the firm’s estimated intrinsic value of the stock. Under this circumstance, firms can use (1) the expected aftermarket stock price as the intrinsic value of the firm, or (2) still keep its original estimate of the intrinsic value but treat the expected high price in the aftermarket as a windfall gain. If the firm revises its estimate of the intrinsic value, all our model implications will still hold.

However, under circumstance (2) where the firm keeps its original estimate of the intrinsic value and treats the possible high price in the aftermarket as a potential gain when the IPO is successful, then the story could be different. Indeed, if we include a firm’s inability to sell its assets in the stock market as a cost when an offer fails, the deadweight cost will be particularly high when the aftermarket is hot. In other words, when issuers believe that their assets can be traded at a much higher price in the stock market than in the product market, they have an incentive to underprice their stocks to avoid the possibility of a failed IPO. On the other hand, if the market is cold and issuers are not sure if their stocks will be traded at their perceived intrinsic values, the issuers might have more incentive to price the stock at the value they believe the stock is worth. Given this, we should observe a lower offering price (relative to the firm’s perceived intrinsic value) during a hot market than during a cold market. Our reasoning complements the explanation offered by Loughran and Ritter (2002). These authors argue that issuers will care less about the offering price of an IPO if they expect their stocks
to perform well in the aftermarket. If we also include the private benefits for insiders of the issuing companies (such as family programs and executive stock option programs, which are frequently used tools during the internet frenzy) as a cost if an IPO fails, then the issuer (at least the executives of the issuing firm) might have more incentive to lower the offering price. This is especially true if the strike price of the executive option is equal to the offering price of the IPO (rather than to the first aftermarket price). 21

In addition, when the market is hot (and flooded with many IPOs), it is possible that the search cost, \( c_3 \), and the estimated range of investor’s post-search values \( a \) will also be reduced. According to our model, a firm will have more incentive to offer a low price when the magnitudes of \( a \) and \( c_3 \) are small. Given this, all three parameters in our model indicate that IPOs in a hot market should be more underpriced than if otherwise.

Since our model predictions are based on three parameters \( W_i \), \( a \), and \( c_3 \), it is quite possible that firms in the same industry might share same parameter values. For example, the deadweight cost should be about the same within an industry. Consequently, firms in industries with more intangible assets have an incentive to price their IPOs lower than firms in industries with more tangible assets. (The amount of intangible assets can be used as a proxy for the deadweight costs \( W_i \) when an IPO fails.) It can also be argued that \( a \) and \( c_3 \) might be high for industries in the early stage. As the industry matures, the magnitudes of \( a \) and \( c_3 \) should be reduced. Given this, holding everything else constant, industries at different stages of maturity might have different magnitudes of search costs \( (c_3) \) and estimated ranges of post-search values \( (a) \) for their IPOs. Benveniste, Busaba, and Wilhelm (2002) and Benveniste, Ljungquist, Wilhelm, and Yu (2003) believe that valuation uncertainty is composed of an industry component and a firm-specific component. It will be less costly and easier for investors to evaluate the value of a firm if there is a lot of information on other firms in the same industry. On the other hand, for firms in industries that do not have much information (or comparables) for investors to adequately assess their values (and, therefore, have high estimated ranges of post-search values) and do not have much deadweight costs when an IPO failed, their IPO stocks may be aggressively sold to investors at prices higher than firms’ perceived intrinsic stock values.

The Optimal Underwriting Contract

Our model so far ignores the role played by an underwriter and can only be applied to a situation described by a best-efforts contract (although in this contract issuers still need an underwriter). However, if we assume that the underwriter maximizes the joint payoff to the issuer and the underwriter, maximizing the joint payoff is the same as maximizing the issuer’s payoff (since the underwriter does not need to buy all the stocks from the issuer if the IPO fails). Therefore, we did not bring the underwriter into the picture explicitly in the previous sections.
In this section, we will introduce an underwriter into our model and analyze a firm commitment contract. To simplify the model development and to adhere to the spirit of our model, we assume that the underwriter and the firms share the same information and have identical goals. In other words, there are neither information asymmetry nor agency conflicts between these two parties. Since the objectives of the firm and the underwriter are assumed to be the same, the optimal pricing decision is the one that maximizes the joint payoff to the firm and the underwriter.

To start the analysis, we assume that the underwriter will purchase the IPO share at price $p_i$ if the offer fails. When this happens, the underwriter will sell the share in the aftermarket, while the firm will receive the offering price $p_i$ from the underwriter. The underwriting costs are sunk and are not considered in the model. We also ignore the commission disparity between a best-efforts and a firm commitment contract by assuming a unique commission rate $m_i$ for all types of IPOs.

Therefore, if the stock is successfully sold in the aftermarket at price $p_i$, the issuing firm’s payoff is $p_i(1 - m_i)$ and its underwriter’s payoff is $p_i m_i$. The joint payoff to the firm and its underwriter is $p_i$. If the issuance fails, the firm’s payoff is still the same $p_i(1 - m_i)$ since the underwriter will purchase the stock from the issuer. (Under a best-efforts contract, the firm receives $W_i$ and its underwriter receives zero.) The underwriter will then sell the stock in the aftermarket at $M_i$. (Our assumptions about the firms and their underwriters imply that they both have the same estimate of the aftermarket price $M_i$.) In addition, if the offer fails, we assume that the underwriter faces a reputation cost (for not being able to sell the stock in the IPO market and will consequently lose market share). We term this the underwriter’s failure cost $\sigma_i$. Given this, the payoff to the underwriter when the offer fails is $M_i - \sigma_i - p_i(1 - m_i)$. Correspondingly, the joint payoff to the firm and the underwriter is $M_i - \sigma_i$. In our model framework, the joint payoffs to the firm and underwriter are the same (regardless of contract type) if a firm is not searched by an investor. Exhibit 7 summarizes the possible payoffs to the underwriter and the firm for the ten terminal nodes identified in Exhibit 2.

**Firm’s Strategies Under a Firm Commitment Contract**

The optimal pricing strategy of a firm using a firm commitment contract can be solved by following the procedure we used above. The results are summarized in Proposition 2.

**Proposition 2.** With a firm commitment contract, when firm $F_i$ observes $V_i - a \leq \bar{V}_j + \xi - 2\sqrt{c_2} \xi \leq V_i + a$ (case 1), its optimal pricing strategy will result in $D_i^* = V_i - p_i^* = 3a - c_3 - \sqrt{2}a(6a - V_i - 2c_3 + M_i - \sigma_i)$. When firm $F_i$ observes $V_i - a > \bar{V}_j + \xi - 2\sqrt{c_2} \xi$ (case 2), its optimal pricing strategy will result in $D_i^* = V_i - p_i^* = \frac{1}{2}(V_i - a - c_3 - M_i + \sigma_i)$. When firm $F_i$ observes
The price decreases in the underwriter’s failure cost.

Estimated range of the post-search values, and the expected market clearance price.

Suggest that the IPO offering price increases in the investor’s search costs.

\( D \)

\( V \)

\( p \)

\( i \)

\( j \)

\( c \)

\( \sigma \)

\( V_i - \rho_i - c_i, p_i, V \)

\( -c_i - c_2, M_i - \sigma_i, V \)

\( V_i - \rho_i - c_i - c_2, p_i, \overline{M}_2 - \sigma_2 \)

\( V_i - \rho_i - c_i - c_2, \overline{M}_i - \sigma_i, p \)

\( V_i - \rho_i - c_i - c_2, \overline{M}_i - \sigma_i, \overline{M}_2 - \sigma_2 \)

\( V_i - p_i - c_i, V, p_i \)

\( -c_i - c_2, V, \overline{M}_2 - \sigma_2 \)

\( V_i - p_i - c_i - c_2, p_i, \overline{M}_2 - \sigma_2 \)

\( V_i - p_i - c_i - c_2, \overline{M}_i - \sigma_i, p \)

\( -c_i - c_2 - c_3, \overline{M}_i - \sigma_i, \overline{M}_2 - \sigma_2 \)

\( V_j + \xi - 2 \sqrt{c_2 \xi} > V_i + a \) (case 3), its optimal pricing strategy will result in

\( D^*_i = V_i - p^*_i = 3a - c_3 - 2 \sqrt{a}(3a - V_i - c_3 + \overline{M}_i - \sigma) \). All the equilibria suggest that the IPO offering price increases in the investor’s search costs, the estimated range of the post-search values, and the expected market clearance price.

The price decreases in the underwriter’s failure cost.

\( \frac{\partial p^*_i}{\partial c_3} > 0 \) (which implies \( \frac{\partial p^*_i}{\partial c_i} > 0 \) and \( \frac{\partial p^*_i}{\partial \gamma} > 0 \)), \( \frac{\partial p^*_i}{\partial a} > 0 \),

and \( \frac{\partial p^*_i}{\partial \sigma_i} < 0 \). (14)
The signs take opposite directions when we analyze the difference between the firm’s intrinsic value and the offering price ($D^*_i = V_i - p_i^*$), or

$$\frac{\partial D^*_i}{\partial c_3} < 0 \text{ (which implies } \frac{\partial D^*_i}{\partial c_1} < 0 \text{ and } \frac{\partial D^*_i}{\partial Y} < 0), \frac{\partial D^*_i}{\partial a} < 0,$$

and $\frac{\partial D^*_i}{\partial \sigma_i} > 0$. (15)

**Proof.** See the Appendix.

The results resemble those under the best-efforts contract, with the downside payoff $W_i$ being replaced by $M_i - \sigma_i$. Again, the relationship between $D^*_i$ and $a$ and the relationship between $D^*_i$ and $c_3$ are exactly the same as that reported for the best-efforts contract. However, now when the issuance fails (i.e., the offered stocks cannot be sold at the IPO stage), the cost to the issuers and the underwriter combined is changed from $V_i - W_i$ into $V_i - (M_i - \sigma_i)$. If we assume the firm’s perceived intrinsic value of its stock (or $V_i$) is the same as the price it estimates the stock will be traded in the open market (or $M_i$), the deadweight costs when an IPO fails will be $\sigma_i$. Equation (14) reports that firms will lower the IPO price if the underwriter failure cost $\sigma_i$ is high. With this simplification, all implications derived in Proposition 1 hold in Proposition 2.

**Contract Selection**

The firm and the underwriter will select an underwriting contract that generates a higher joint payoff $E(\Pi_i)$ to them. Under this circumstance, since the two contracts differ only in the downside payoffs, the contract selection is actually the selection of a better downside payoff. We define $\Lambda_i$ as the downside payoff, where $\Lambda_i = W_i$ if a best-efforts contract is selected and $\Lambda_i = M_i - \sigma_i$ if a firm commitment contract is selected. Given this simplification, the contract selection problem becomes:

$$\max_{\Lambda_i \in \{W_i, M_i - \sigma_i\}} E(\Pi_1) = \frac{1}{2} E(\Pi_1(\Lambda_i))_L + \frac{1}{2} E(\Pi_1(\Lambda_i))_R \quad (16)$$

Substituting the optimal price in each case derived in Propositions 1 and 2 into the corresponding expected payoff function, we can compare the expected payoffs under the two contracts. The one with the higher expected payoff is the one that should be selected by the firm. Proposition 3 summarizes the firm’s optimal contract decision rules.
Proposition 3. When firm $F_i$ and its underwriter maximize their joint payoff, the optimal contract for them is (1) a firm commitment contract if $M_i - \sigma_i > W_i$, (2) a best-efforts contract if $M_i - \sigma_i < W_i$, and (3) either contract if $M_i - \sigma_i = W_i$.

Proof. See the Appendix.

The result indicates that the underwriting contract choice depends only on the magnitudes of the downside payoffs. This result seems intuitive. Since the underwriter and the owner maximize their joint payoff and since the two contracts differ only in the downside payoff, both the underwriter and the owner will select the contract with a higher downside payoff (or a lower failure cost). When the cost of a failed IPO is higher for the firm than for the underwriter (or $V_i - W_i > \sigma_i$), they will select a best-efforts contract. However, if the underwriter’s failure cost is higher than that of the firm’s (or $\sigma_i > V_i - W_i$), then they will select a firm commitment contract.

We recognize that the set-up of our model in this section is over-simplified and ignores a rich literature on the roles underwriters play in the IPO process. However, our sole purpose in this section is to demonstrate that if the underwriter and the owner jointly maximize their payoff, the results of our model presented in the previous section still hold (including an underpricing of the IPO stock).

Actually, without appealing to asymmetric information and other related stories, our result (based on deadweight and search costs) supports Muscarella and Vetsuypens’s (1989) finding that underwriters underprice their own IPOs as much as other IPOs in the market during a similar period. When an underwriter underwrites its own stocks, it must maximize the joint payoff to both the owner and the underwriter.

Conclusion

This study is an attempt to provide a novel, yet convincing, model to explain the IPO pricing puzzle reported in the vast empirical literature. We approach this issue from the owner’s perspective, and model a firm’s pricing strategies conditional upon an investor’s decision-making process. We suggest that (1) the deadweight cost (reputation loss and issuance cost) for an issuing firm if its issuance fails, (2) the additional information search costs for an investor if alternative investments are investigated, and (3) the likelihood that investors can get consensus opinion (or draw from a narrow range) on the value of the IPO stock, are the three parameters affecting a firm’s pricing decision. Therefore, depending on the parameter values, an IPO can be overpriced, slightly underpriced, or significantly underpriced. Our model is particularly suitable for explaining the anomalous evidence reported for REIT IPOs, MLP IPOs, and mutual fund IPOs (for which current theories fail to provide a full explanation). In addition, our results provide a partial explanation for the IPO hot-market phenomenon and suggest that IPO underpricing levels could vary across industries.
Our model framework can be extended to include the interactions between the underwriter and the issuer. Although an equilibrium solution for a model with this type of extension might be very difficult to derive, we believe that a successful attempt in this direction should generate insights that help us further understand the puzzling IPO phenomena. Furthermore, our model can be revised to explain the pricing decisions in other product (or financial) markets. The results of our model seem to be helpful in guiding the pricing decision of a residential property owner. Similar to the situation posited in our model, owners in a residential property market also face competition from their neighbors and a buyer may have to incur certain costs to search for other properties before making a purchase.

Appendix

Proof for Lemma 1

The investor will pay additional search cost to look for investment opportunities other than the two stocks. Taking into account this cost, if \( \hat{V}_i - p_i \geq -c_3 \), the investor knows for sure one of the two stocks will be purchased (but still does not know if it will be firm \( F_i \) or firm \( F_j \)). If \( \hat{V}_i - p_i < -c_3 \), the investor knows for sure that stock of \( F_i \) will not be purchased at \( p_i \) (but does not know whether to accept \( p_j \) yet).

When \( \hat{V}_i - p_i \geq -c_3 \)

After the first search (and with a realized value \( \hat{V}_i \)), the investor needs to decide whether to conduct a second search. When \( \hat{V}_i - p_i \geq -c_3 \), the investor knows there are two choices. First, the investor can accept \( p_i \) without a second search. Second, the investor can conduct a second search on the stock value of the second firm and then decide on whether to accept \( p_i \) or \( p_j \). To make a decision, the investor needs to compare the payoff of accepting \( p_i \) with the expected payoff of conducting a second search. To calculate the latter, the investor has to first estimate the expected payoff of the second firm’s stock. Since the investor does not know \( \hat{V}_j \) but knows the second IPO stock value’s pre-search distribution \( V_{bj} \sim U[\hat{V}_j - \xi, \hat{V}_j + \xi] \), the expected payoff from a second search depends on the offering prices \((p_i \text{ and } p_j) \) set by the firms, or:

\[
E(II_j) = \int_{V_j - \xi}^{\hat{V}_j - p_i + p_j} (\hat{V}_i - p_i) dV_{bj} + \int_{\hat{V}_j - p_i + p_j}^{\hat{V}_j + \xi} (V_{bj} - p_j) dV_{bj} - c_1 - c_2. \tag{A1}
\]
This expected payoff of conducting a second search can then be compared with the investor’s payoff from accepting $p_i$ without a second search ($NA_i$), which is $\hat{V}_i - p_i - c_i$. The difference is:

$$
\begin{align*}
(V_i - p_i - c_i) & - \left( \int_{\hat{V}_j - \xi}^{\hat{V}_i - p_i + p_i} (\hat{V}_i - p_i) dV_{b_j} + \int_{\hat{V}_j - p_i + p_i}^{\hat{V}_j + \xi} (\hat{V}_b - p_j) dV_{b_j} - c_2 - c_j \right) \\
& = -\frac{1}{4\xi} (\hat{V}_i - p_i + p_j - \hat{V}_j - \xi)^2 + c_2.
\end{align*}
$$

(A2)

The investor should (should not) conduct a second search if Equation (A2) is negative (positive). This means that the investor should not conduct the second search if $\hat{V}_i - p_i > \hat{V}_j + \xi - p_j + 2\sqrt{c_2\xi}$ and conduct the second search if $\hat{V}_i - p_i < \hat{V}_j + \xi - p_j - 2\sqrt{c_2\xi}$. Note that if the investor is willing to pass up the second search, it means that the profit from investing in the first stock, $\hat{V}_i - p_i$, must be high enough. Given this, we can eliminate the condition $\hat{V}_i - p_i \leq \hat{V}_j + \xi - p_j + 2\sqrt{c_2\xi}$. If an investor needs to conduct a second search, we know that the profit from investing in the first stock ($\hat{V}_i - p_i$) is not high enough. Given this, we can eliminate the condition $\hat{V}_i - p_i > \hat{V}_j + \xi - p_j + 2\sqrt{c_2\xi}$. With these two simplifications, the decision rule can be simplified to:

- pass up the second search iff $\hat{V}_j + \xi - p_j - 2\sqrt{c_2\xi} \leq \hat{V}_i - p_i$,
- and $\hat{V}_j + \xi - p_j - 2\sqrt{c_2\xi} > \hat{V}_i - p_i$.

(A3)

Combining with the first condition $\hat{V}_i - p_i \geq -c_3$, Equation (A3) indicates that if $\hat{V}_i - p_i \leq -c_3$ and $\hat{V}_j + \xi - p_j - 2\sqrt{c_2\xi} \leq \hat{V}_i - p_i$, the investor will accept $p_i$ without conducting a second search. The investor will accept the payoff at terminal node $NA_i$. If $\hat{V}_i - p_i \geq -c_3$ and $\hat{V}_j + \xi - p_j - 2\sqrt{c_2\xi} > \hat{V}_i - p_i$, the investor conducts a second search. After this search is done, the investor will compare $\hat{V}_j - p_j$ with $\hat{V}_i - p_i$ to make the final decision. The investor will select terminal node $SA_i$ (accepting $p_i$ after the second search) iff $\hat{V}_i - p_i \geq \hat{V}_j - p_j$. The investor will accept terminal node $SR_i$ (accepting $p_j$ after the second search) iff $\hat{V}_j - p_j < \hat{V}_i - p_i$. 
When $\hat{V}_i - p_i < -c_3$

After the first search, when the investor observes $\hat{V}_i - p_i < -c_3$, the investor knows that the stock will not be purchased at $p_i$. The next decision the investor needs to make is whether a second search should be conducted to investigate the value of the second stock. The investor knows that if there is a second search, the result will be either accepting $p_j$ (terminal node $SR_i$, with a payoff $V_{bj} - p_j$) or rejecting both (terminal node $SO_i$, with a payoff $-c_3$). Again, since the stock value of the second firm before the search is $V_{bj} \sim U[\bar{V}_j - \xi, \bar{V}_j + \xi]$, the expected value of conducting a second search depends on the offering price of the second firm $p_j$ and the magnitude of the third search costs $c_3$, or:

$$E(II_j) = \int_{\bar{V}_j - \xi}^{p_j - c_3} (-c_3)dV_{bj} + \int_{p_j - c_3}^{\bar{V}_j + \xi} (V_{bj} - p_j)dV_{bj} - c_1 - c_2.$$ 

This expected payoff of conducting a second search can be compared to the investor’s payoff from rejecting both offers without a second search ($NO_i$), which is $-c_1 - c_3$. The difference is:

$$(-c_1 - c_3) - \left( \int_{\bar{V}_j - \xi}^{p_j - c_3} (-c_3)dV_{bj} + \int_{p_j - c_3}^{\bar{V}_j + \xi} (V_{bj} - p_j)dV_{bj} - c_1 - c_2 \right) = -\frac{1}{4\xi} (p_j - c_3 - \bar{V}_j - \xi)^2 + c_2.$$

The investor should (should not) conduct a second search if this equation is negative (positive). This means that the investor should not conduct the second search if $\bar{V}_j + \xi - 2\sqrt{c_2\xi} + c_3 \geq p_j$ (or $p_j \geq \bar{V}_j + \xi + 2\sqrt{c_2\xi} + c_3$), and conduct the second search if $\bar{V}_j + \xi - 2\sqrt{c_2\xi} + c_3 < p_j < \bar{V}_j + \xi + 2\sqrt{c_2\xi} + c_3$. Note that if the investor is willing to pass up the second search, it means that the second offering price $p_j$ must be high enough. Therefore, we can eliminate the condition $\bar{V}_j + \xi - 2\sqrt{c_2\xi} + c_3 \geq p_j$. If an investor needs to conduct a second search, we know that the second offering price $p_j$ is not high enough. Given this, we can eliminate the condition $\bar{V}_j + \xi - 2\sqrt{c_2\xi} + c_3 < p_j$. With these two simplifications, the decision rule can be simplified to:
pass up the second search iff \( p_j \geq \bar{V}_j + \xi + 2\sqrt{c_2\xi} + c_3 \), and
conduct the second search iff \( p_j < \bar{V}_j + \xi + 2\sqrt{c_2\xi} + c_3 \). (A4)

Combining with the first condition \( \hat{V}_i - p_i < -c_3 \), Equation (A4) indicates if \( \hat{V}_i - p_i < -c_3 \) and \( p_j > \bar{V}_j + \xi + 2\sqrt{c_2\xi} + c_3 \), the investor will reject both offers without a second search and the game will end at \( NO_i \). However, if \( \hat{V}_i - p_i < -c_3 \) and \( p_j < \bar{V}_j + \xi + 2\sqrt{c_2\xi} + c_3 \), the investor will conduct a second search and obtain a realized \( \hat{V}_j \). The investor will select terminal node \( SR_i \) (accepting \( p_j \), with a payoff \( \hat{V}_j - p_j - c_1 - c_2 \)) iff \( \hat{V}_j - p_j \geq -c_3 \). The investor will select terminal node \( SO_i \) (rejecting both offers, with a payoff \( -c_1 - c_2 - c_3 \)) that gives the payoff iff \( \hat{V}_j - p_j < -c_3 \).

This provides the comprehensive decision rules for the investor on the left branch of the game tree (see Exhibit 1). The decision rules on the right branch of the game tree are similar to that for the left branch, except that the first firm being searched is \( F_j \). End of proof.

**Proof for Proposition 1**

To prove Proposition 1, we first examine the boundary conditions of all the 24 situations summarized in Exhibit 5. The first step is to identify those situations that cannot exist because of a conflicting criterion in establishing the orders. We find nine situations (situations 1, 3, 5, 9, 11, 13, 17, 19, and 21) with conflicting constraints. After deleting these nine situations, we analyze the optimal solutions for the remaining fifteen situations. We obtain an interior solution for the optimal price for situations 4, 14, and 20. We obtain a corner optimal solution for situations 2, 6, 10, 12, and 18. However, we also find that these five corner situations are dominated by the situations with an interior solution. Although situations 7, 8, 15, 16, 22, 23, and 24 have a solution, careful analyses indicate that those local solutions cannot be the global optimal solutions because of deadweight costs. We will compare the results of all the 24 situations to select the optimal solutions for the firms.

For simplicity and without loss of generality, we assume that \( 2a > V_i - W_i + c_3 \). This sets the maximum social welfare loss when an IPO issuance fails to \( 2a \). We know that when an IPO fails, the firm’s loss is \( V_i - W_i \) and the investor’s loss is \( c_3 \). This makes sense, as the total loss should not be greater than the difference between the maximum price \( (V_i + a) \) and minimum price \( (V_i - a) \) an investor will pay. Again, we also assume that firms are symmetric in their pricing decisions and firms with identical characteristics should price the same.
Situations with an Interior Solution

We find three situations that have a local optimal solution, namely, situations 4, 14, and 20. To be a local optimal solution, the optimal price derived must also satisfy the boundary conditions that characterize the situation. In the interest of saving space, we will provide a detailed proof for situation 4 and shorter proofs for the other two situations.

Situation 4. Situation 4 requires $A_i \leq B_i \leq C_i \leq D_i$ and $p_j < \psi$. From Panel B of Exhibit 4, we can write the expected payoff function of $F_i$ in this situation as:

\[
E(II)_L = \int_{V_i+p_j}^{V_i+p_j+p_j} (p_j) \frac{1}{2a} d\hat{V}_i + \int_{p_j-c_3}^{V_i+p_j+p_j} \left( \int_{V_j}^{V_j+p_j} (p_i) \frac{1}{2a} d\hat{V}_j \right) \frac{1}{2a} d\hat{V}_j
\]

\[
+ \int_{p_j-c_3}^{p_j-c_3} (p_i) \frac{1}{2a} d\hat{V}_i.
\]

(A5)

and

\[
E(II)_R = \int_{V_i+p_j}^{V_i+p_j+p_j} (V_i) \frac{1}{2a} d\hat{V}_i + \int_{p_j-c_3}^{V_i+p_j+p_j} \left( \int_{V_j}^{V_j+p_j} (p_j) \frac{1}{2a} d\hat{V}_j \right) \frac{1}{2a} d\hat{V}_j
\]

\[
+ \int_{p_j-c_3}^{p_j-c_3} (W_i) \frac{1}{2a} d\hat{V}_i + \int_{p_j-c_3}^{p_j-c_3} (p_j) \frac{1}{2a} d\hat{V}_j.
\]

(A6)

Equation (A5) characterizes the situation where the investor’s search starts with firm $F_i$. If the first search reveals that $\hat{V}_i < V_j + \xi + p_i - p_j - 2\sqrt{c_2}\xi$, the investor will go along with route [1]—immediately accepts $p_i$ and firm $F_i$ receives $p_{\text{opt}}$. If the first search reveals that $\hat{V}_i \geq V_j + \xi + p_i - p_j - 2\sqrt{c_2}\xi \geq p_i - c_3$, the investor will go along with route [2]—conducts the second search and compares $\hat{V}_i - p_j$ with $\hat{V}_j - p_j$. The payoff of this route is reported in the third row of column 2 in Panel B of Exhibit 4. Finally, if the first search reveals that $\hat{V}_i \geq [V_i - a, p_i - c_3]$, we have $p_i - c_3 > \hat{V}_i$. The investor will go along with route...
conducts the second search and compares \( \hat{V}_j - p_j \) with \(-c_3\). The payoff of \( F_i \) in this route is \( W_i \), regardless of the outcome.

Equation (A6) characterizes the situation where the investor’s search starts with firm \( F_2 \). If the first search reveals that \( \hat{V}_j \in [\hat{V}_i + \xi + p_j - p_i - 2\sqrt{c_2\xi}, V_j + a] \), we have \( \hat{V}_j \geq \hat{V}_i + \xi + p_j - p_i - 2\sqrt{c_2\xi} \geq p_j - c_3 \). Based on the decision rule in Exhibit 4, the investor will go along with route [1]—immediately accepts \( p_j \) and firm \( F_i \) receives \( V_i \). If the first search reveals that \( \hat{V}_j \in [p_j - c_3, \hat{V}_i + \xi + p_j - p_i - 2\sqrt{c_2\xi}] \), we have \( p_j - c_3 \leq \hat{V}_j < \hat{V}_i + \xi + p_j - p_i - 2\sqrt{c_2\xi} \). The investor will go along route [2]—conducts the second search and compares \( \hat{V}_i - p_i \) with \( \hat{V}_j - p_j \). The payoff of this route is specified in the third row of column 3 in Panel B of Exhibit 4. Finally, if the first search reveals that \( \hat{V}_j \in [V_j - a, p_j - c_3] \), we have \( \hat{V}_j < p_j - c \). The investor will go along with route [4]—conducts the second search and compares \( \hat{V}_j - p_j \) with \(-c_3 \). The payoff of \( F_i \) in this route is reported in the 5th row of column 3 in Panel B of Exhibit 4.

The optimization problem for \( F_i \) is to:

\[
\text{Max } E(\Pi_i) = \frac{1}{2} E(\Pi_i)_L + \frac{1}{2} E(\Pi_i)_R. \tag{A7}
\]

The first-order condition for \( F_i \) is:

\[
\frac{dE(\Pi_i)}{dp_i} = -\frac{1}{16a^2} \{-6a^2 + 2c_3^2 - 3p_i^2 - 4c_3p_j \\
+ 4p_j^2 + p_j + 2(p_i - p_j)W_i - 4V_i(p_j - c_3) \\
+ 2V_i^2 - 4a(c_3 - 4p_i + p_j + W_i + 2V_i)\} = 0. \tag{A8}
\]

Similarly, the optimization problem of \( F_j \) is to:

\[
\text{Max } E(\Pi_j) = \frac{1}{2} E(\Pi_j)_L + \frac{1}{2} E(\Pi_j)_R. \tag{A9}
\]
and the first-order condition for $F_j$ is:

$$
\frac{dE(II)}{dp_j} = -\frac{1}{16a^2} \{ -6a^2 + 2c_3^2 - 3p^2_j - 4c_3 p_i + 4p_i p_j \\
+ p_i + 2(p_j - p_i)W_j - 4V_i(p_i - c_3) + 2V_j^2 \\
- 4a(c_3 - 4p_j + p_i + W_j + 2V_j) \} = 0.
$$

(A10)

From Equations (A8) and (A10), we know that when $V_i = V_j$ and $W_i = W_j$, $p_i$ must equal $p_j$. Since we assume that firms are symmetric in their pricing decisions, this means that firms with the same value parameters must be priced the same.

Substituting $p_i = p_j$ into Equation (A8), we obtain:

$$
\frac{1}{8a^2} \{ -(p_i - V_i - c_3 + 3a)^2 + 2a(6a - V_i - 2c_3 + W_j) \} = 0.
$$

(A11)

Since $6a - V_i - 2c_3 + W_i = 3(2a - c_3) - V_i + W_i + c_3 > (2a - c_3) - V_i + W_i + c_3 > 0$, we have an interior solution. In addition, since $p_i - V_i - c_3 + 3a = ((p_i - c_3) - (V_i - a)) + 2a \geq 0 + 2a > 0$, the interior solution is unique. Equation (A11) can be simplified to:

$$
p_i^* = p_j^* = -3a + V_i + c_3 + \sqrt{2a(6a - V_i - 2c_3 + W_i)}. \quad (A12)
$$

Since:

$$
\frac{d^2E(II)}{dp_i} = -\frac{1}{8a^2} (8a - 3p_i + 2p_j + W_j) \\
< -\frac{1}{8a^2} (8a - (V_i + a) - c_3 + W_j) \\
(given \ p_i - c_3 < V_i + a) \\
< -\frac{1}{8a^2} (6a - V_i - 2c_3 + W_i) < 0,
$$
we know that the solution is a local maximum.

However, to prove that \( p_i^* \) is the optimal solution for this situation, we also need to confirm that \( p_i^* \) satisfies all the constraints that characterize situation 4. In other words, we must show that \( p_i^* \) is consistent with all the constraints when \( V_i - a \leq \bar{V}_j + \xi - 2\sqrt{c_2}\xi \leq V_i + a \) holds. This can be done by demonstrating:

\[
\begin{align*}
\mathbf{D}_i - \mathbf{C}_i &= (V_i + a) - (\bar{V}_j + \xi + p_i - p_j - 2\sqrt{c_2}\xi) \\
&= (V_i + a) - (\bar{V}_j + \xi - 2\sqrt{c_2}\xi) \geq 0, \quad (A13) \\
\mathbf{C}_i - \mathbf{B}_i &= (\bar{V}_j + \xi + p_i - p_j - 2\sqrt{c_2}\xi) - (p_i - c_3) \\
&= (\bar{V}_j + \xi - 2\sqrt{c_2}\xi) \\
&\quad - (-3a + V_i + \sqrt{2a(6a - V_i - 2c_3 + W_j)}) \\
&= (\bar{V}_j + \xi + 3a - V_i - 2\sqrt{c_2}\xi) \\
&\quad - \sqrt{2a(6a - V_i - 2c_3 + W_j)} \geq 0 \quad (A14)
\end{align*}
\]

(given \( W_i < V_i \) and \( \bar{V}_j + \xi \geq V_j + a = V_i + a \),

\( \bar{V}_j + \xi + 3a - V_i > 4a \),

\[
\begin{align*}
\mathbf{B}_i - \mathbf{A}_i &= (p_i - c_3) - (V_i - a) \\
&= (-3a + V_i + \sqrt{2a(6a - V_i - 2c_3 + W_j)}) - (V_i - a) \\
&= \sqrt{2a(6a - V_i - 2c_3 + W_j)} - 2a \\
&= \sqrt{2a} \cdot \sqrt{4a - V_i - 2c_3 + W_i} \\
&\geq \sqrt{2a} \cdot \sqrt{2(a - c_3) - V_i + W_i} \geq 0, \quad (A15)
\end{align*}
\]

\[
\begin{align*}
p_j - \psi &= (-3a + V_j + c_3 + \sqrt{2a(6a - V_j - 2c_3 + W_j)}) \\
&\quad - (\bar{V}_j + \xi + 2\sqrt{c_2}\xi + c_3) \\
&= -3a - (\bar{V}_j + \xi) - 2\sqrt{c_2}\xi + V_j \\
&\quad + \sqrt{2a(6a - V_j - 2c_3 + W_j)} < 0 \quad (A16)
\end{align*}
\]

(given \( W_j < V_j \) and \( \bar{V}_j + \xi \geq V_j + a \),

\[-3a - (\bar{V}_j + \xi) + V_j \leq -4a \).

When \( V_i - a \leq \bar{V}_j + \xi - 2\sqrt{c_2}\xi \leq V_i + a \), Equations (A13), (A14), (A15) and (A16) demonstrate that the optimal interior solution \( p_i^* \) also satisfies parameter
conditions \( A_i \leq B_i \leq C_i \leq D_i \) and \( p_j < \psi \) (that characterize situation 4). This indicates that Equation (A12) is the local optimal solution for situation 4.

**Situation 14.** Situation 14 requires that \( C_i \leq A_i \leq B_i \leq D_i \) and \( p_j < \psi \). From Panel B of Exhibit 4, we can write the expected payoff function of \( F_i \) in this situation as:

\[
E(\Pi)_L = \int_{p_i-c_3}^{V_i+a} (p_i) \frac{1}{2a} d\hat{V}_i + \int_{p_i-c_3}^{V_i} (W_i) \frac{1}{2a} d\hat{V}_i \tag{A17}
\]

and

\[
E(\Pi)_R = \int_{p_j-c_3}^{V_j+a} (V_i) \frac{1}{2a} d\hat{V}_j \\
+ \int_{p_j-c_3}^{p_i-c_3} \left( \int_{p_j-c_3}^{V_i+a} (W_i) \frac{1}{2a} d\hat{V}_i + \int_{p_i-c_3}^{V_i+a} (p_j) \frac{1}{2a} d\hat{V}_j \right) \frac{1}{2a} d\hat{V}_j. \tag{A18}
\]

The optimization problem for \( F_i \) is to:

\[
\max_{(p_i)} E(\Pi) = \frac{1}{2} E(\Pi)_L + \frac{1}{2} E(\Pi)_R. \tag{A19}
\]

The first-order condition for \( F_i \) is:

\[
\frac{dE(\Pi)}{dp_i} = \frac{1}{8a^2} (-2p_i + a + V_i + W_i + c_3(p_j + 3a - V_j - c_3)) \\
= 0. \tag{A20}
\]

Since:

\[
p_j + 3a - V_j - c_3 = (p_j - c_3) - (V_j - a) + 2a \geq 2a > 0, \tag{A21}
\]
we have an interior solution:

\[ p_i = \frac{1}{2} (a + V_i + c_3 + W_i). \quad (A22) \]

Since:

\[
\frac{d^2E(II)}{dp_i} = -\frac{1}{4a^2} (p_j - c_3 - (V_j - a) + 2a) < 0 \quad (A23)
\]

(given \( p_j - c_3 \geq V_j - a \)),

the solution is a local maximum. Similarly, we can derive that:

\[ p_j = \frac{1}{2} (a + V_j + c_3 + W_j). \quad (A24) \]

However, to prove that \( p_j^* \) is the optimal solution for this situation, we also need to confirm that \( p_j^* \) satisfies all the constraints that characterize situation 14 when \( V_i - a \geq \bar{V}_j + \xi - 2\sqrt{c_2}\xi \) holds. This can be done by demonstrating:

\[
\begin{align*}
\mathbf{D}_j - \mathbf{B}_i &= (V_i + a) - \left( \frac{1}{2} (a + V_i + c_3 + W_i) - c_3 \right) \geq 0, \quad (A25) \\
\mathbf{B}_i - \mathbf{A}_i &= (p_i - c_3) - (V_i - a) > 0, \quad (A26) \\
\mathbf{A}_i - \mathbf{C}_i &= (V_i - a) - (\bar{V}_j + \xi + p_i - p_j - 2\sqrt{c_2}\xi) > 0, \quad (A27) \\
p_j - \psi &= \frac{1}{2} (a + V_i + c_3 + W_i) - (\bar{V}_j + \xi + 2\sqrt{c_2}\xi + c_3) < 0 \quad (A28)
\end{align*}
\]

(given \( \bar{V}_j + \xi \geq V_j + a = V_i + a \) and \( \bar{V}_j + \xi \geq V_i \geq W_i \)).

When \( V_i - a \geq \bar{V}_j + \xi - 2\sqrt{c_2}\xi \), Equations (A25), (A26), (A27), and (A28) demonstrate that the optimal interior solution \( p_j^* \) also satisfies parameter conditions \( \mathbf{C}_i \leq \mathbf{A}_i \leq \mathbf{B}_i \leq \mathbf{D}_i \) and \( p_j < \psi \) that characterize situation 14. This indicates that Equation (A23) is the local optimal solution for situation 14.
Situation 20. Situation 20 requires that \( A_i \leq B_i \leq D_i \leq C_i \) and \( p_j < \psi \). From Panel B of Exhibit 4, we can write the expected payoff function of \( F_i \) in this situation as:

\[
E(\Pi_i)_L = \int_{p_i-c_3}^{V_i+a} \left( \int_{V_i-a}^{V_j+a} (p_i) \frac{1}{2a} \, d\hat{V}_i + \int_{V_i-a}^{V_j+a} (p_j) \frac{1}{2a} \, d\hat{V}_j \right) \frac{1}{2a} \, d\hat{V}_i + \int_{V_i-a}^{p_j-c_3} (W_i) \frac{1}{2a} \, d\hat{V}_i \tag{A29}
\]

and

\[
E(\Pi_i)_R = \int_{p_j-c_3}^{V_j+a} \left( \int_{V_j-a}^{V_i+a} (W_i) \frac{1}{2a} \, d\hat{V}_i + \int_{V_j-a}^{V_i+a} (p_i) \frac{1}{2a} \, d\hat{V}_i \right) \frac{1}{2a} \, d\hat{V}_j + \int_{V_j-a}^{p_i-c_3} (W_j) \frac{1}{2a} \, d\hat{V}_j \tag{A30}
\]

The optimization problem for \( F_i \) is to:

\[
\text{Max}_{\{p_i\}} E(\Pi_i) = \frac{1}{2} E(\Pi_i)_L + \frac{1}{2} E(\Pi_i)_R. \tag{A31}
\]

The first-order condition for \( F_i \) is:

\[
\frac{dE(\Pi_i)}{dp_i} = -\frac{1}{16a^2} \left\{ -6a^2 + 2c_3^2 - 3p_i^2 - 4c_3p_j + 4p_dp_j + p_j + 2(p_i - p_j)W_i - 4V_i(p_j - c_3) + 2V_i^2 - 4a(c_3 - 4p_i + p_j + 2W_i + V_i) \right\} = 0. \tag{A32}
\]

We could derive a similar condition for \( F_j \). Again, since the two firms have symmetric pricing decisions and same value parameters, they should be priced the same. Substituting \( p_i = p_j \) into Equation (A32), we obtain:
Since \(3a - V_i - c_3 + W_i = 2a - V_i - c_1 + W_i + a > 0\), the interior solution for Equation (A33) does exist. In addition, since \(p_i - V_i - c_3 + 3a = ((p_i - c_3) - (V_i - a)) + 2a \geq 0 + 2a > 0\), the interior solution is unique:

\[
p_i = -3a + V_i + c_3 + 2\sqrt{a(3a - V_i - c_3 + W_i)}. \tag{A34}
\]

Since:

\[
\frac{d^2E(\Pi)}{dp_i^2} = -\frac{1}{8a^2}(8a - 3p_i + 2p_j + W_j) < 0 \tag{A35}
\]

(given \(p_i - c_3 < V_i + a\))

the solution is a local maximum. Similarly, we can derive that:

\[
p_j = -3a + V_j + c_3 + 2\sqrt{a(3a - V_j - c_3 + W_j)}. \tag{A36}
\]

However, to prove that \(p_i^*\) is the optimal solution for this situation, we also need to confirm that \(p_i^*\) satisfies all the constraints that characterize situation 14 when \(\overline{V}_j + \xi - 2\sqrt{c_2\xi} > V_i + a\) holds. This can be done by demonstrating:

\[
\begin{align*}
C_i - D_i &= (\overline{V}_j + \xi - 2\sqrt{c_2\xi}) - (V_i + a) > 0, \tag{A37} \\
D_i - B_i &= (V_i + a) - (p_i - c_3) \\
&= 2\sqrt{a} \cdot \sqrt{a} + V_i + c_3 - W_i \geq 0, \tag{A38} \\
B_i - A_i &= (p_i - c_3) - (V_i - a) \\
&= 2\sqrt{a} \cdot \sqrt{2a - V_i - c_3 + W_i} \geq 0, \tag{A39} \\
p_j - \psi &= -3a - (\overline{V}_j + \xi) - 2\sqrt{c_2\xi} \\
&+ V_j + 2\sqrt{a(3a - V_j - c_3 + W_j)} < 0 \tag{A40} \\
&\text{(given } \overline{V}_j + \xi \geq V_j + a \text{ and } W_j < V_j). 
\end{align*}
\]
When $\bar{V}_j + \xi - 2\sqrt{c_2\xi} > V_i + a$, Equations (A37), (A38), (A39), and (A40) demonstrate that the optimal interior solution $p_i^*$ also satisfies parameter conditions $A_i \leq B_i \leq D_i \leq C_i$ and $p_j < \psi$ that characterize situation 20. This indicates that Equation (A34) is the local optimal solution for situation 20.

**Situations with a Corner Solution**

We find five situations with a corner solution, namely, situations 2, 6, 10, 12, and 18. For those situations, we will check if they are dominated by adjoining interior situations.

**Situation 2.** This situation requires that $B_i \leq A_i \leq C_i \leq D_i$ and $p_j < \psi$. From Panel B of Exhibit 4, we can write the expected payoff function of $F_i$ in this situation as:

$$E(\Pi)_L = \int_{V_j}^{V_i+a} (p_j) \frac{1}{2a} d\bar{V}_j + \int_{V_i-a}^{V_j+p_j} (W_i) \frac{1}{2a} d\bar{V}_i$$

and

$$E(\Pi)_R = \int_{V_i}^{V_j+a} (V_j) \frac{1}{2a} d\bar{V}_j + \int_{V_j-a}^{V_i+p_j} (p_i) \frac{1}{2a} d\bar{V}_i$$

(A41)

The optimization problem for $F_i$ is to:

$$\text{Max}_{\{p_i\}} E(\Pi) = \frac{1}{2} E(\Pi)_L + \frac{1}{2} E(\Pi)_R.$$  

(A43)
The first-order condition for $F_i$ is:

$$
\frac{dE(II)}{dp_i} = \frac{1}{16a^2} (8a^2 - (p_i - p_j)(3p_i - p_j - 2W_i) \\
+ 4a(-2p_i + p_j + W_i)). \tag{A44}
$$

Similarly, the first-order condition for $F_j$ is:

$$
\frac{dE(II)}{dp_j} = \frac{1}{16a^2} (8a^2 - (p_j - p_i)(3p_j - p_i - 2W_j) \\
+ 4a(-2p_j + p_i + W_j)). \tag{A45}
$$

From Equations (A44) and (A45), we know that when $V_i = V_j$ and $W_i = W_j$, $p_i$ must equal $p_j$. This reduces Equation (A44) to $dE(II)/dp_i = 1/4a (-p_i + 2a + W_i)$. Given $p_i - c_3 \leq V_i - a$ (i.e., $p_i \leq V_i - a + c_3$), this indicates:

$$
\frac{dE(II)}{dp_i} \geq \frac{1}{4a} (3a - V_i - c_3 + W_i) > 0. \tag{A46}
$$

Equation (A46) indicates that the expected payoff of $F_i$ is an increasing function of $p_i$. Given the constraint $B_i \leq A_i \leq C_i \leq D_i$ and $p_j < \psi$, Equation (A46) suggests that the optimal price must be at the point where $B_i$ (or, $p_i - c_3$) is equal to its upper limit $A_i$. Given this, it is clear that this optimal solution must also be the point that is adjacent to the lower bound of another situation. It is clear that situation 4 (where $A_i \leq B_i \leq C_i \leq D_i$) is the adjoining situation. Since situation 4 has an interior solution, this corner solution must be dominated by the interior solution.

Finally, we also need to check if the corner solution satisfies the constraints $B_i \leq A_i \leq C_i \leq D_i$ and $p_j < \psi$. Note that $B_i = A_i$ leads to $p_i - c_3 = V_i - a$, hence $p_i = V_i - a + c_3 < V_j + \xi + 2\sqrt{c_2\xi} + c_3 = \psi$. In other words, constraint $p_j < \psi$ is supported by the corner solution. In addition, the constraint $A_i \leq C_i \leq D_j$ is satisfied when the parameter condition $V_i - a \leq V_j + \xi - 2\sqrt{c_2\xi} \leq V_i + a$ holds.
Situation 6. Situation 6 requires that $A_i \leq C_i \leq B_i \leq D_i$ and $p_j < \psi$. From Panel B of Exhibit 4, we can write the expected payoff function of $F_i$ in situation 6 as:

$$E(II)_L = \int_{p_i-c_3}^{V_i+a} (p_i) \frac{1}{2a} d\hat{V}_i + \int_{V_i-a}^{p_i-c_3} (W_i) \frac{1}{2a} d\hat{V}_i$$

(A47)

and

$$E(II)_R = \int_{p_j-c_3}^{V_j+a} (V_i) \frac{1}{2a} d\hat{V}_j$$

$$+ \int_{V_j-a}^{p_j-c_3} \left( \int_{V_i-a}^{p_i-c_3} (W_i) \frac{1}{2a} d\hat{V}_i + \int_{V_i-a}^{V_i+a} (p_j) \frac{1}{2a} d\hat{V}_i \right) \frac{1}{2a} d\hat{V}_j.$$  

(A48)

The optimization problem for $F_i$ is to:

$$\text{Max} \ E(II) = \frac{1}{2} E(II)_L + \frac{1}{2} E(II)_R.$$  

(A49)

The first-order condition for $F_i$ is:

$$\frac{dE(II)}{dp_i} = \frac{1}{8a^2} (-2p_i + a + V_i + W_i + c_3)(p_j + 3a - V_j - c_3).$$  

(A50)

It is easy to see that $p_j + 3a - V_j - c_3 = (p_j - c_3) - (V_j - a) + 2a \geq 2a > 0$. We also know that $-2p_i + a + V_i + W_i + c_3 < -2(\hat{V}_i + \xi - 2\sqrt{c_2\xi} + c_3) + a + V_i + W_i + c_3 = -(\hat{V}_i + \xi - (a + V_i)) + (\hat{V}_i + \xi + c_3 - 4\sqrt{c_2\xi} - W_i) < 0$. This is true because when $C_i \leq B_i$, $\hat{V}_i + \xi + p_j - p_i - 2\sqrt{c_2\xi} \leq p_j - c_3$. Given this:

$$\frac{dE(II)}{dp_i} < 0.$$  

(A51)

Equation (A51) indicates that the expected payoff of $F_i$ is a decreasing function of $p_i$. Given the constraint $A_i \leq C_i \leq B_i \leq D_i$, Equation (A51) suggests that the
optimal price must be at the point where $B_i$ (or, $p_i - c_3$) is equal to its lower limit $C_i$. Given this, it is clear that this optimal solution must also be the point that is adjacent to the upper bound of another situation. It is clear that situation 4 (where $A_i \leq B_i \leq C_i \leq D_i$) is the adjoining situation. Since situation 4 has an interior solution, this corner solution must be dominated by the interior solution.

Finally, we also need to check if the corner solution satisfies the constraints $A_i \leq B_i \leq C_i \leq D_i$ and $p_j < \psi$. We know that $p_i = \bar{V}_j + \xi + c_3 - 2\sqrt{c_2\xi} < \bar{V}_j + \xi + c_3 + 2\sqrt{c_2\xi} = \psi$. In addition, the constraint $A_i \leq C_i \leq D_i$ is satisfied when the parameter condition $V_i - a \leq \bar{V}_j + \xi - 2\sqrt{c_2\xi} \leq V_i + a$ holds.

**Situations 10 and 12.** Situation 10 requires that $B_i \leq C_i \leq A_i \leq D_i$ and $p_j < \psi$. Situation 12 requires $C_i \leq B_i \leq A_i \leq D_i$ and $p_j < \psi$. Both situations require that $B_i, C_i \leq A_i = \min(\bar{V}_i) \leq \bar{V}_i$. From Exhibit 4, we know that route [1] is the only feasible outcome under this circumstance.

From Panel B of Exhibit 4, we can write the expected payoff function of $F_i$ in these situations as:

$$E(\Pi_i) = \frac{1}{2} E(\Pi_i)_L + \frac{1}{2} E(\Pi_i)_R = \frac{1}{2} (p_i + V_i).$$

The first-order condition for $F_i$ is:

$$\frac{dE(\Pi_i)}{dp_i} = \frac{1}{2} > 0. \quad (A52)$$

Equation (A52) indicates that the expected payoff of $F_i$ for situations 10 and 12 is an increasing function of $p_i$.

Given the constraint $B_i \leq C_i \leq A_i \leq D_i$ that is required for situation 10, Equation (A52) suggests that the optimal price must be at the point where $B_i$ (or, $p_i - c_3$) is equal to its upper limit $C_i$. Given this, it is clear that this optimal solution must also be the point that is adjacent to the lower bound of another situation. It is clear that situation 12 (where $C_i \leq B_i \leq A_i \leq D_i$) is the neighboring situation. Since situation 12 has a corner solution, this corner solution of situation 10 must be dominated by the corner solution of situation 12. Given the constraint $C_i \leq B_i \leq A_i \leq D_i$ that is required for situation 12, Equation (A51) suggests that the optimal price must be at the point where $B_i$ (or, $p_i - c_3$) is equal to its upper limit $A_i$. Therefore, it is clear that this optimal solution must also be the point that is adjacent to the lower bound of another situation. It is clear that situation 14 (where $C_i \leq A_i \leq B_i \leq D_i$) is the adjoining situation. Since situation 14 has an interior solution, this corner solution of situation 12 and hence the corner solution of situation 10 must be dominated by the interior solution.
Finally, we also need to check if the corner solutions satisfy their constraints. In situation 10, \( B_i = C_i \) leads to \( p_i - c_3 = \tilde{V}_j + \xi - 2\sqrt{c_2\xi} \), hence \( p_i = \tilde{V}_j + \xi + c_3 - 2\sqrt{c_2\xi} < \tilde{V}_j + \xi + c_3 + 2\sqrt{c_2\xi} = \psi \). In situation 12, \( B_i = A_i \) leads to \( p_i - c_3 = \tilde{V}_i - a \), hence \( p_i = (\tilde{V}_i - a) + c_3 \leq \tilde{V}_j + \xi + c_3 < \psi \). Consequently, constraint \( p_j < \psi \) is supported by both corner solutions. The common constraint \( C_i \leq A_i \) for both situations is satisfied when the parameter condition \( \tilde{V}_j + \xi - 2\sqrt{c_2\xi} \leq V_i - a \) holds.

**Situation 18.** This situation requires \( B_i \leq A_i \leq D_i \leq C_i \) and \( p_j < \psi \). From Panel B of Exhibit 4, we can write the expected payoff function of \( F_i \) in situation 18 as:

\[
E(II)_L = \int_{V_i-a}^{V_i+a} \left( \int_{V_j-a}^{V_j+p_i+p_j} \frac{1}{2a} d\tilde{V}_j \right) d\tilde{V}_i + \frac{1}{2a} d\tilde{V}_j,
\]

and

\[
E(II)_R = \int_{V_i-a}^{V_i+a} \left( \int_{V_j-a}^{V_j+p_i+p_j} (W_i) \frac{1}{2a} d\tilde{V}_j \right) d\tilde{V}_i + \int_{V_j-a}^{V_j+p_i+p_j} \left( \frac{1}{2a} d\tilde{V}_i \right)
\]

The first-order condition for \( F_i \) is:

\[
\frac{dE(II)}{dp_i} = \frac{1}{2a} (-2p_i + p_j + a + W_j).
\]

Similarly, the first-order condition for \( F_j \) is:

\[
\frac{dE(II)}{dp_j} = \frac{1}{2a} (-2p_j + p_i + a + W_i).
\]

From Equations (A55) and (A56), we know that when \( V_i = V_j \) and \( W_i = W_j \), \( p_i \) must equal \( p_j \). This reduces Equation (A55) to

\[
\frac{dE(II)}{dp_i} = 1/4a (-p_i + 2a + W_j).
\]

Given \( p_i - c_3 \leq V_i - a \) (i.e., \( p_i \leq V_i - a + c_3 \)), this indicates:
Equation (A57) indicates that the expected payoff of $F_i$ is an increasing function
of $p_i$. Given the constraint $B_i \leq A_i \leq D_i \leq C_i$, Equation (A57) suggests that
the optimal price must be at the point where $B_i$ (or, $p_i - c_3$) is equal to its upper limit
$A_j$. Given this, it is clear that this optimal solution must also be the point that is
adjacent to the lower bound of another situation. It is clear that situation 20 (where $A_i \leq B_i \leq D_i \leq C_i$) is the adjoining situation. Since situation 20 has an interior
solution, this corner solution must be dominated by the interior solution.

Finally, we also need to check if the corner solution satisfies the constraints $B_i \leq A_i \leq D_i \leq C_i$ and $p_j < \psi$. We note that $B_i = A_i$ leads to $p_i - c_3 = V_i - a$. Therefore, $p_i = V_i - a + c_3 < \bar{V}_j + \xi + 2\sqrt{c_2 \xi} + c_3 = \psi$. The constraint $D_i \leq C_i$ is satisfied when the parameter condition $V_i - a \leq \bar{V}_j + \xi - 2\sqrt{c_2 \xi}$ holds.

**Situations with Conflicting Constraints**

Nine situations have problems meeting all the constraints imposed upon them. For
the purpose of discussion, we divide these nine situations into two groups.

**Situations 1, 3, 5, 9, 11, 13, 17, and 19.** These eight situations have the common
requirements that $B_i \leq A_i \leq D_i \leq C_i$ and $p_j \leq \psi$. Since the constraint $p_j \leq \psi$ indicates $p_i \geq \psi$, $p_i \geq \bar{V}_j + \xi + 2\sqrt{c_2 \xi} + c_3$. Therefore, we have:

\[
p_i - c_3 \geq \bar{V}_j + \xi + 2\sqrt{c_2 \xi} > \text{Max}(\bar{V}_i)
\]

(A58)

Equation (A58) implies that $B_i > D_i$. Since $B_i \leq D_i$ and $B_i > D_i$ cannot exist
simultaneously, these 8 situations do not exist in reality.

**Situation 21.** This situation requires that $A_i \leq D_i \leq B_i \leq C_i$ and $p_j \leq \psi$ (and
$A_j \leq D_j \leq B_j \leq C_j$ and $p_i \geq \psi$). Since the constraint $p_j \geq \psi$ indicates $p_i \geq \psi$,
$p_i \geq \bar{V}_j + \xi + 2\sqrt{c_2 \xi} + c_3$. Thus, we have:

\[
p_i - c_3 \geq \bar{V}_j + \xi + 2\sqrt{c_2 \xi} > \bar{V}_j + \xi - 2\sqrt{c_2 \xi}.
\]

(A59)

Equation (A59) implies that $B_i > C_i$. Since $B_i \leq C_i$ and $B_i > C_i$ cannot exist
simultaneously, this situation does not exist in reality.
**Situations with a Deadweight Loss**

We find seven situations with deadweight loss, namely, situations 7, 8, 15, 16, 22, 23, and 24. There is no optimal solution within each of these.

**Situations 8, 16, 22, and 24.** These four situations have the common requirements that \( B_i \geq D_i \) and \( p_j < \psi \). With these two constraints, Panel A of Exhibit 4 indicates that three routes are not feasible for the investor. Since \( B_i \geq D_i = \max(V_i) \geq \hat{V}_i \), routes [1] and [2] are not feasible solutions for the investor. Since \( p_j < \psi \), the condition of route [3] in Exhibit 4 cannot be satisfied. Consequently, the only feasible route is route [4]. Since the payoff of route [4] is the downside payoff \( W_i \) for both firms, the two firms will set their prices at a level that allow them to avoid the four situations. In other words, in reality, there is no optimal price in the four situations.

**Situations 7, 15, and 23.** These three situations have the common requirements that \( D_i \leq B_i \) and \( p_j \geq \psi \). With these two constraints, Panel A of Exhibit 4 indicates that three routes are not feasible for the investor. Since \( B_i \geq D_i = \max(\hat{V}_i) \geq \hat{V}_i \), routes [1] and [2] are not feasible solutions for the investor. Since \( p_j \geq \psi \), the condition of route [4] in Exhibit 4 cannot be satisfied. Consequently, the only feasible route is route [3]. From Exhibit 4, we know that the investor will immediately reject both offers and the payoffs of the two firms are \( \Pi^{[3]}_{il} = W_i \) and \( \Pi^{[3]}_{ir} = V_i \), respectively. The expected payoff will be \( E(\Pi_i) = \frac{1}{2} E(\Pi_{il}) + \frac{1}{2} E(\Pi_{ir}) = \frac{1}{2} (W_i + V_i) \). This indicates a positive expected deadweight loss \( V_i - \frac{1}{2} (W_i + V_i) \). Consequently, both firms will set their prices at a suitable level to avoid taking a negative payoff. In other words, in reality, there is no optimal price in the three situations.

**Summary of All 24 Situations**

There are a total of four types of outcomes for the 24 situations. First, situations 4, 14, and 20 have an interior solution. Second, situations 2, 6, 10, 12, and 18 have a corner solution. However, those corner solutions are dominated by the adjoining interior solutions. Third, situations 1, 3, 5, 9, 11, 13, 17, 19, and 21 have conflicting boundary constraints that cannot simultaneously hold. These situations do not exist. Fourth, situations 8, 16, 22, 24, 7, 15, and 23 suffer positive expected deadweight loss. Firms will definitely set the price at a right level to avoid those situations. In reality, these situations do not exist.

Exhibit 6 summarizes the results. First, when parameter condition \( A_i \leq C_i \leq D_i \) (that is, \( V_i - a \leq \hat{V}_j + \xi - 2\sqrt{c_3\xi} \leq V_i + a \)) holds, there are eight possible outcomes (situations 1 to 8). It is clear that the global optimum within these eight situations is the interior solution in situation 4 \( (p^*_i = -3a + V_j + c_j + \sqrt{2a(6a - V_j - 2c_j + W_j)}) \), which is proven to dominate the corner solutions of
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situations 2 and 6. Second, when parameter condition $C_i < A_i$ (that is, $V_i - a > V_j + \xi - 2\sqrt{c_2\xi}$) holds, there are 8 possible outcomes (situations 9 to 16). It is clear that the global optimum within these eight situations is the interior solution in situation 14 ($p_i^* = \frac{1}{2} (a + V_i + c_3 + W_i)$), which is proven to dominate the corner solutions of situations 10 and 12. Finally, when parameter condition $C_i > D_i$ (that is, $V_j + \xi - 2\sqrt{c_2\xi} > V_i + a$) holds, there are eight possible outcomes (situations 17 to 24). It is clear that the global optimum within these eight situations is the interior solution in situation 20 ($p_i^* = -3a + V_i + c_3 + 2\sqrt{a(3a - V_i - c_3 + W_i)}$), which is proven to dominate the corner solution of situation 18. It is also clear that these three cases represent the comprehensive solution set for firms’ IPO pricing strategies.

Comparative Static Analysis

When $D_i^* = V_i - p_i^* = 3a - c_3 - \sqrt{2a(6a - V_i - 2c_3 + W_i)}$ (Case 1), we obtain:

$$\frac{\partial D_i^*}{\partial c_3} = -1 + \frac{2a}{\sqrt{2a(2a + 4a - V_i - 2c_3 + W_i)}} < 0$$

(given $4a > V_i + 2c_3 - W_i$),

$$\frac{\partial D_i^*}{\partial c_1} = \frac{\partial D_i^*}{\partial c_3} \cdot \frac{\partial c_3}{\partial c_1} = \gamma^2 \frac{\partial D_i^*}{\partial c_3} < 0$$

(given $c_3 = c_1 \gamma^2$),

$$\frac{\partial D_i^*}{\partial \gamma} = \frac{\partial D_i^*}{\partial c_3} \cdot \frac{\partial c_3}{\partial \gamma} = 2\gamma c_1 \frac{\partial D_i^*}{\partial c_3} < 0,$$

$$\frac{\partial D_i^*}{\partial W_i} = -\frac{a}{\sqrt{2a(6a - V_i - 2c_3 + W_i)}} < 0,$$

$$\frac{\partial D_i^*}{\partial a} = 3 - \frac{12a - V_i - 2c_3 + W_i}{\sqrt{2a(6a - V_i - 2c_3 + W_i)}}$$

$$= \frac{\sqrt{18a(6a - V_i - 2c_3 + W_i)} - (12a - V_i - 2c_3 + W_i)}{\sqrt{2a(6a - V_i - 2c_3 + W_i)}}$$

$$< 0$$

(A60)
When $D_i^* = V_i - p_i^* = \frac{1}{2} (V_i - a - c_3 - W_i)$ (Case 2), we obtain:

$$\frac{\partial D_i^*}{\partial c_3} = -\frac{1}{2} < 0,$$

$$\frac{\partial D_i^*}{\partial c_1} = \frac{\partial D_i^*}{\partial c_3} \cdot \frac{\partial c_3}{\partial c_1} = \gamma^2 \frac{\partial D_i^*}{\partial c_3} < 0 \text{ (given } c_3 = c_1 \gamma^2),$$

$$\frac{\partial D_i^*}{\partial \gamma} = \frac{\partial D_i^*}{\partial c_3} + \frac{\partial c_3}{\partial \gamma} = 2 \gamma c_1 \frac{\partial D_i^*}{\partial c_3} < 0,$$

$$\frac{\partial D_i^*}{\partial W_i} = -\frac{1}{2} < 0,$$

$$\frac{\partial D_i^*}{\partial a} = -\frac{1}{2} < 0. \tag{A61}$$

When $D_i^* = V_i - p_i^* = 3a - c_3 - 2\sqrt{a(3a - V_i - c_3 + W_i)}$ (Case 3), we obtain:

$$\frac{\partial D_i^*}{\partial c_3} = -1 + \frac{a}{\sqrt{a(a + 2a - V_i - c_3 + W_i)}} < 0 \text{ (given } 2a > V_i + c_3 - W_i),$$

$$\frac{\partial D_i^*}{\partial c_1} = \frac{\partial D_i^*}{\partial c_3} \cdot \frac{\partial c_3}{\partial c_1} = \gamma^2 \frac{\partial D_i^*}{\partial c_3} < 0 \text{ (given } c_3 = c_1 \gamma^2),$$

$$\frac{\partial D_i^*}{\partial \gamma} = \frac{\partial D_i^*}{\partial c_3} + \frac{\partial c_3}{\partial \gamma} = 2 \gamma c_1 \frac{\partial D_i^*}{\partial c_3} < 0,$$

$$\frac{\partial D_i^*}{\partial W_i} = -\frac{a}{\sqrt{a(3a - V_i - c_3 + W_i)}} < 0,$$

$$\frac{\partial D_i^*}{\partial a} = 3 - \frac{6a - V_i - c_3 + W_i}{\sqrt{a(3a - V_i - c_3 + W_i)}} = \frac{\sqrt{9a(3a - V_i - c_3 + W_i) - (6a - V_i - c_3 + W_i)}}{\sqrt{a(3a - V_i - c_3 + W_i)}} < 0. \tag{A62}$$

The signs for each variable ($c_3, c_1, \gamma, W_i$, and $a$) are consistent among the three cases. End of Proof.
**Proof for Proposition 2**

When analyzing the joint payoffs of the firm and the underwriter, it is easy to see that the firm commitment contract differs from the best-efforts contract only in the downside payoff for the joint party. That is, all we need to do is to replace $W_i$ with $\bar{M}_i - \sigma_i$ in all the equations. The investor’s decision rules reported in Exhibit 3 and the constraints for each of the 24 situations reported in Exhibit 5 remain the same.

Following exactly the approach for deriving Proposition 1, we can derive the pricing rule for a firm commitment contract. Actually, all results are similar to that derived under the best-efforts contract, except that $W_i$ is replaced with $\bar{M}_i - \sigma_i$. The comparative static analysis results can be easily derived. End of Proof.

**Proof for Proposition 3**

Since the joint payoffs (for the firm and the underwriter) of the two underwriting contracts (best-efforts and firm commitment) differ only in the downside payoff, the contract selection is actually a function of the downside payoff $\Lambda_i$, where $\Lambda_i = W_i$ under a best-efforts contract and $\Lambda_i = \bar{M}_i - \sigma_i$ under a firm commitment contract.

**Case $V_i - a > \bar{V}_i + \xi - 2\sqrt{c_2\xi}$**

From the proof of Proposition 1, we know that when firms observe $V_i - a > \bar{V}_i + \xi - 2\sqrt{c_2\xi}$, the optimal price is $p_i^* = \frac{1}{2} (a + V_i + c_3 + \Lambda_i)$. Substituting $p_i^*$ into the firm’s expected payoff function in Equation (A19), we derive:

$$E(\Pi_i) = \frac{1}{64a^2} [7a^3 + a(5c_3^2 - 18c_3(\Lambda_i - V_i) + 13(\Lambda_i - V_i)^2)$$

$$- (c_3 - \Lambda_i + V_i)^2 + a^2(13c_3 + 43\Lambda_i + 21V_i)].$$  \hspace{1cm} (A63)
Correspondingly,

\[
\frac{dE(II)}{d\Lambda_i} = \frac{1}{64a^2} \left[ 43a^2 + 3(c_3 + V_i - \Lambda_i)^2 - 26a(c_3 + V_i - \Lambda_i) + 8ac_3 \right] \\
= \frac{1}{64a^2} \left[ 3 \left( \frac{13}{3} a - (c_3 + V_i - \Lambda_i) \right)^2 - \frac{40}{3} a^2 + 8ac_3 \right] \\
= \frac{1}{64a^2} \left[ 3 \left( \frac{7}{3} a + (2a - c_3 - V_i + \Lambda_i) \right)^2 - \frac{40}{3} a^2 + 8ac_3 \right] \\
> \frac{1}{64a^2} \left[ 3 \left( \frac{7}{3} a \right)^2 - \frac{40}{3} a^2 + 8ac_3 \right] = \frac{1}{64a^2} [3a^2 + 8ac_3] \\
> 0. \quad (A64)
\]

Given that two contracts differ only in the downside payoff, Equation (A64) indicates that a contract with a higher downside payoff will increase the joint payoff of the firm and underwriter. Consequently, a firm will select a firm commitment contract if \( \bar{M}_i - \sigma_i > W_i \), select a best-efforts contract if \( \bar{M}_i - \sigma_i < W_i \), and be indifferent to these two contracts if \( \bar{M}_i - \sigma_i = W_i \).

**Case** \( \bar{V}_i + \xi - 2\sqrt{c_2\xi} > V_i + a \)

When firms observe \( \bar{V}_i + \xi - 2\sqrt{c_2\xi} > V_i + a \), the optimal price is \( p_i^* = -3a + V_i + c_3 + 2\sqrt{a(3a - V_i - c_3 + W_i)} \). Substituting \( p_i^* \) into the firm’s expected payoff function in Equation (A31), we derive:

\[
E(II) = \frac{1}{2a} \left[ 21a^2 + (c_3 + V_i - \Lambda_i)(V_i - \Lambda_i + c_3 + 4\Theta) \right. \\
- \left. 2a(5V_i - 6\Lambda_i + 5c_3 + 6\Theta) \right]. \quad (A65)
\]
where \( \Theta_i = \sqrt{a(3a - V_i - c_3 + \Lambda_i)} \). Correspondingly,

\[
\frac{dE(\Pi_i)}{d\Lambda_i} = \frac{1}{a\Theta_i} \left[ -3\Theta_i^2 + \Theta_i \left( 3a + \frac{\Theta_i^2}{a} \right) \right] \\
= \frac{1}{a^2} \left[ -3a\Theta_i + 3a^2 + \Theta_i^2 \right] \\
= \frac{1}{a^2} \left[ \left( \Theta_i - \frac{3}{2} a \right)^2 + \frac{3}{4} a^2 \right] > 0.
\] (A66)

As in the previous case, a contract with a higher downside payoff will increase the joint payoff of the firm and underwriter. Consequently, a firm will select a firm commitment contract if \( \bar{M}_i - \sigma_i > W_i \), select a best-efforts contract if \( \bar{M}_i - \sigma_i < W_i \), and be indifferent to these two contracts if \( \bar{M}_i - \sigma_i = W_i \).

**Case \( V_i - a \leq \bar{V}_i + \xi - 2\sqrt{c_2} \xi \leq V_i + a \)**

When firms observe \( V_i - a \leq \bar{V}_i + \xi - 2\sqrt{c_2} \xi \leq V_i + a \), the optimal price is \( p_i^* = -3a + \bar{V}_i + c_3 + \sqrt{2a(6a - \bar{V}_i - 2c_3 + \Lambda_i)} \). The total derivative \( dE(\Pi_i)/d\Lambda_i \) can be decomposed as:

\[
\frac{dE(\Pi_i)}{d\Lambda_i} = \frac{\partial E(\Pi_i)}{\partial \Lambda_i} + \frac{\partial E(\Pi_i)}{\partial p_i^*} \cdot \frac{\partial p_i^*}{\partial \Lambda_i}, \quad (A67)
\]

Where:

\[
\frac{\partial E(\Pi_i)}{\partial \Lambda_i} = \frac{1}{8a^2} \left[ 3a^2 + (V_i + c_3 - p_i^*)^2 \right. \\
\left. + 2a(\bar{V}_i + \xi - 2\sqrt{c_2} \xi + p_i^* - c_3 - 2\bar{V}_i) \right] \\
\geq \frac{1}{8a^2} \left[ 3a^2 + (V_i + c_3 - p_i^*)^2 \right. \\
\left. + 2a(V_i - a + p_i^* - c_3 - 2\bar{V}_i) \right] \\
= \frac{1}{8a^2} \left[ (a - c_3 - V_i + p_i^*)^2 + 2ac_3 \right] > 0; \quad (A68)
\]

\[
\frac{\partial p_i^*}{\partial \Lambda_i} = \frac{a}{\sqrt{2a(6a - \bar{V}_i - 2c_3 + \Lambda_i)}} < \frac{a}{\sqrt{2a \cdot 2a}} = \frac{1}{2} > 0. \quad (A69)
\]
Given Equations (A67), (A68), and (A69), it is clear that if \( \frac{\partial E(II)}{\partial p_i^*} \geq 0 \), we must have \( \frac{dE(II)}{d\Lambda_i} > 0 \).

However, even if \( \frac{\partial E(II)}{\partial p_i^*} < 0 \), we can still prove that \( \frac{dE(II)}{d\Lambda_i} > 0 \). To see this, we know if \( \frac{\partial E(II)}{\partial p_i^*} < 0 \), since \( \frac{\partial p_i^*}{\partial \Lambda_i} < \frac{1}{2} \), we can rewrite Equation (A67) as:

\[
\frac{dE(II)}{d\Lambda_i} = \frac{\partial E(II)}{\partial \Lambda_i} + \frac{\partial E(II)}{\partial p_i^*} \cdot \frac{\partial p_i^*}{\partial \Lambda_i} > \frac{\partial E(II)}{\partial \Lambda_i} + \frac{1}{2} \cdot \frac{\partial E(II)}{\partial p_i^*} \tag{A70}
\]

\[
= \frac{1}{16a^2} [9a^2 + 2a(2(\bar{V}_j + \xi) - 2\sqrt{c_2\xi}) - 3V_i + \Lambda_i - c_3) + (V_i + c_3 - p_i^*)(p_i^* + c_3 + V_i - 2\Lambda_i)]
\]

\[
> \frac{1}{16a^2} [9a^2 + 2a(2(V_i - a) - 3V_i + \Lambda_i - c_3) + (V_i + c_3 - p_i^*)(p_i^* + c_3 + V_i - 2\Lambda_i)]
\]

\[
= \frac{1}{16a^2} [2a(2a - (V_i - \Lambda_i + c_3)) + a^2 + (V_i - c_3 - p_i^*)(p_i^* + c_3 + V_i - 2\Lambda_i)]
\]

\[
> \frac{1}{16a^2} [a^2 + (V_i + c_3 - p_i^*)(p_i^* + c_3 + V_i - 2\Lambda_i)].
\]

(A71)

Since \( p_i^* + c_3 + V_i - 2\Lambda_i > 0 \) is always true, if \( V_i + c_3 - p_i^* \geq 0 \), Equation (88) must hold and \( dE(II)/d\Lambda_i > 0 \). However, if \( V_i + c_3 - p_i^* < 0 \) (which can be transformed into \( 1.5a > V_i - \Lambda_i + 2c_3 \)), we can still show that \( dE(II)/d\Lambda_i > 0 \) if we can prove \( p_i^* + c_3 + V_i - 2\Lambda_i < 3a \) and \( 0 > V_i + c_3 - p_i^* > -a/3 \).

First, to prove \( p_i^* + c_3 + V_i - 2\Lambda_i < 3a \) is equivalent to prove \( \sqrt{2a(6a - V_i - 2c_3 + \Lambda_i)} < 6a - 2c_3 - 2V_i + 2\Lambda_i \). We know:

\[
(6a - 2c_3 - 2V_i + 2\Lambda_i)^2 - 2a(6a - V_i - 2c_3 + \Lambda_i)
\]

\[
= [2(3a - (V_i - \Lambda_i + c_3)) - 0.5a]^2 + 2ac_3 - 6.25a^2
\]

\[
> [2(1.5a) - 0.5a]^2 + 2ac_3 - 6.25a^2
\]

(given that \( 1.5a > V_i - \Lambda_i + 2c_3 \) for this situation)

\[
= 2ac_3 > 0.
\]

(A72)
Given Equation (A72), $2a(6a - V_i - 2c_3 + \Lambda_i) < (6a - 2c_3 - 2V_i + 2\Lambda_i)^2$ must hold and $p_i^* + c_3 + V_i - 2\Lambda_i < 3a$ is true.

Second, to prove $V_i + c_3 - p_i^* > -a/3$ is equivalent to prove $\frac{10}{3}a > \sqrt{2a(6a - V_i - 2c_3 + \Lambda_i)}$. After several transformations, this is equivalent to prove $V_i - \Lambda_i + 2c > \frac{4}{3}a$, which is intuitively true. Thus, $0 < p_i^* + c_3 + V_i - 2\Lambda_i < 3a$ and $0 > V_i + c_3 - p_i^* > -a/3$ hold in this situation. We can now rewrite Equation (A71) as:

\[
\frac{dE(\Pi_i)}{d\Lambda_i} > \frac{1}{16a^2} \left[ a^2 + (V_i + c_3 - p_i^*)(p_i^* + c_3 + V_i - 2\Lambda_i) \right] \\
> \frac{1}{16a^2} \left[ a^2 - \frac{1}{3} a \cdot 3a \right] = 0. \tag{A73}
\]

We now prove that $dE(\Pi_i)/d\Lambda_i > 0$ holds in this case. Given that the best-efforts and firm commitment contracts differ only in the downside payoff, Equation (A73) indicates that a contract with the higher downside payoff will increase the joint payoff of the firm and underwriter. Consequently, a firm should select a firm commitment contract if $\bar{M_i} - \sigma_i > W_i$, select a best-efforts contract if $\bar{M_i} - \sigma_i < W_i$, and be indifferent to these two contracts if $\bar{M_i} - \sigma_i = W_i$.

End of Proof.

Endnotes

1 We obtained this information from an article written by Jay Ritter titled “Some Factoids about the 2006 IPO Market,” which is available on his website, http://bear.cba.ufl.edu/ritter.


4 To explain the dot-com bubble in the IPO market, Loughran and Ritter (2002, 2004), Ljungqvist and Wilhelm (2003), James and Karceski (2006), and Houston, James, and
Karceski (2006) stress the dark side of the institutional arrangements that boost the underwriter’s profit, affect analyst coverage, and increase the personal gains of executives of the IPO firms.

5 See Chan, Stohs, and Wang (2001) and Chan, Erickson, and Wang (2002) for empirical evidence on REIT IPOs, Muscarella (1988) and Michaely and Shaw (1994) for empirical evidence on MLP IPOs, and Peavy (1990) for empirical evidence on mutual fund IPOs.

6 We assume that the two firms simultaneously make offers to the market so that we do not need to model who should move first, although, in reality, later movers have the advantage to observe the prices of the earlier movers before they make decisions. The one share assumption is not necessary for the model development, but simplifies our model presentation greatly.

7 Our result is not sensitive to the assumption that each firm knows its rival’s stock value. If we assume that firms do not know each other’s value, the qualitative conclusions of the model are substantially the same. All we need to do is to replace a known value with an expected value in the analyses, which does not affect the rest of the analyses.

8 The prices offered by the firms are the IPO offering prices, not the ranges of preliminary prices offered during the road shows.

9 Here we implicitly assume that the search will yield information that can justify the search cost. We can allow the investor to choose whether or not to conduct the first search. However, this will make the model more complicated while not generating more implications.

10 The key assumption we need here is that the search must refine the information in such a way that the post-search $\hat{V}_k$ is on average closer to the intrinsic value $V_k$ than the pre-search value $\bar{V}_k$. Given this, it does not matter if the mean of $\hat{V}_k$ differs from (or is equal to) $V_k$.

11 Alternatively, we can treat the search costs $c_1$ and $c_2$ as sunk costs that cannot be recovered. The key is that the investor will incur more costs when searching for other investment opportunities.

12 Learning can be achieved due to the high correlations of future cash flows among firms with a similar background. See Foerster and Karolyi (1999) for a good discussion on this issue. However, Kaustia and Knupfer (2008) find that personal experience is overweighted when compared to rational Baysian learning. Wang, Chan, and Erickson (1995), Wang, Erickson, Gau, and Chan (1995), Chan, Leung, and Wang (2005), and Daniels and Phillips (2007) report the relationship between REIT performance and the attention the stock receives from the capital market. Li and Wang (1995) and Ling (2005) discuss the predictability of REIT stock returns and commercial real estate returns, respectively.

13 The IPO withdrawal rate remained high during the best years for IPOs. Ljungqvist and Wilhelm (2003) report that the IPO withdrawal rate was around 25% during the 1996–2000 period. Stoughton, Wong, and Zechner (2001) suggest that an important motivation of going public is to establish image and publicity in the product market. Given this motivation, a failed IPO will be very costly to a firm. Edelen and Kadlec (2005) also suggest that rational issuers select an offer price that weighs the benefit of higher proceeds against the cost of forgone surplus if the offer fails.

14 It can be argued that the payoff to the investor should be $-c_1 - c_3$ because the investor will not search and there is no learning. However, our result is the same either way. The $-c_1 - c_3$ specification makes the presentation more manageable.
Note that we assume that the intrinsic value of the stock $V_{bx}$ ($x = i, j$) follows a uniform distribution $U[\bar{V}_x - \xi, \bar{V}_x + \xi]$. 

This 24-situation scenario assumes that firms will use the same optimal price strategies and will not deliberately play games against their rivals (which will lead a firm to make a decision that deviates from the optimal one). In other words, as we mentioned before, we assume that firms are symmetric in making pricing decisions. If we allow the rival firms to play games, a firm with a 50% chance of being searched first needs to consider $2^4 \times 2^4 = 576$ possible situations.

See Ling and Ryngaert (1997) for a good discussion on the change of REIT structure during the period.

It should noted that $a = 0.05V$ means that investors’ judgments could differ within 10% of the firm’s value. This is quite reasonable, at least in the product (or property) market. We frequently observe that in a sealed bid auction, the range of the bids is within $\pm 10\%$ of the mean of the bids. This happens despite the fact that all the parties involved are knowledgeable and with similar information.

For a discussion on IPO waves and the associated return patterns, see Pastor and Pietro (2006).

We stress again that this conclusion assumes that the firm’s perceived intrinsic value is independent of the expected aftermarket stock price. The firm will, however, treat the expected high aftermarket price as a windfall gain if the IPO is successful. Otherwise, this conclusion will not hold, at least within our model framework.

See Ljungqvist and Wilhelm (2003) for empirical evidence related to the private benefits of listing.

We know that we miss a significant part of IPO literature by making this assumption. However, our purpose in this section is to demonstrate that the implications of our model still hold with a firm commitment contract. We will not be able to examine the interactions between the issuer and the underwriter.

In the best-efforts case, we assume that the investor will conduct the search after observing the offering prices. In this firm commitment case, we can change the assumption to one in which the investor conducts the search before observing the offering prices. The result is the same.

Implicitly, we assume the underwriting costs to be the same across different contracts. In practice, Menon and Williams (1991) show that a firm commitment contract is more expensive than a best-efforts contract due to the need to hire more credible auditors.

However, it should be noted that the underwriter and the issuers can collude to maximize their joint payoff only when the expected $\bar{M}$ is the same as the intrinsic value of the firm $V$ that they both agree on. When the expected $\bar{M}$ differs from $V$, the objectives of the underwriter and the issuer cannot be the same and our analyses (based on the maximization of the joint payoff) should not be applied to those cases.

References


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